Fundamentals of fluid dynamics and examples

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Flow regime

- Classification of flow by compressibility :

- Compressible flows
- Incompressible flows
- (weakly compressible flows)

- Some other classifications:

- Friction: Flows w/wo friction, turbulent and laminar flows
- Mach number: Subsonic, transsonic, supersonic, hypersonic
- Behavior: Stationary/instationary flow

Differentiation compressible - incompressible

- Equation for density (isentropic expansion from ideal state 0):

$$\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{\gamma - 1}}$$

- The Mach number determines the possible density variations:

$M \to 0:$ No density variation possible, **incompressible flow**

$M > 0 \;:$ Density variation possible, **compressible flow**

Distinction weakly compressible - compressible - incompressible

- Although density variations (compressibility) become possible for M > 0, todays compressible methods can only work efficiently up to a minimum Mach number of M ≈ 0,1 – 0,3.

- Beneath this threshold the flow can be well considered incompressible. The relative error for density at the Mach number M = 0,3 is ca. 4,4%.

- There is no such thing as incompressible flow but depending on the needed precision, the assumption of incompressibility can be justified.

The Mach number

Definition:

$$M = \frac{v}{c}$$

Relation between flow velocity and speed of sound.
The flow velocity is an easy comprehensible quantity, but what about the speed of sound?

$$c^2 = \left.\frac{dp}{d\rho}\right|_{S=const}$$

A measure, how variations in pressure and density correlate.
A large c results in a small density variation, that correlates with a large variation in pressure.

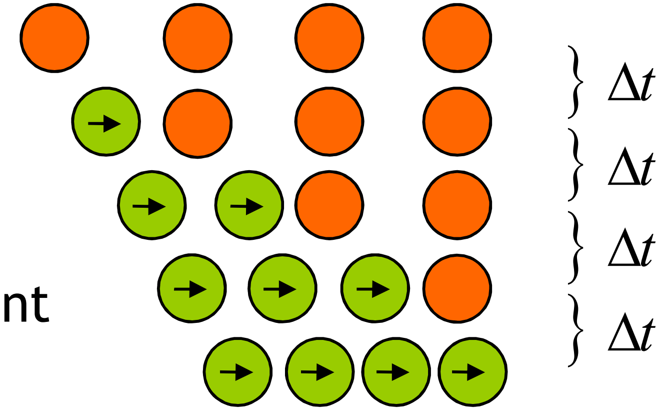
The speed of sound

What does this correlation entail in the context of fluid flow ?

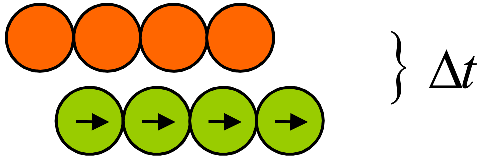
A very simplified model illustrates:

C small: lower density
Change in pressure (disturbance)
Causes change in density

The disturbance spreads from element to element



C → ∞: high density
Change in pressure (disturbance)
Causes no change in density



The disturbance spreads immediately to all elements

Incompressible flows

Fluid flows without or very little density variations arise in a multitude of applications.

That applies in general for liquids. The speed of sound is very high and compression of the medium becomes only possible with very high pressure. The Mach number will therefore be always small.

For slow fluid flows ($M < 0{,}1 – 0{,}3$) the assumption of incompressibility can be justified for some applications. The error, that is introduced by neglecting the pressure variations, is considerably small.

Example for incompressible flows (1)

Flow around a fish under water

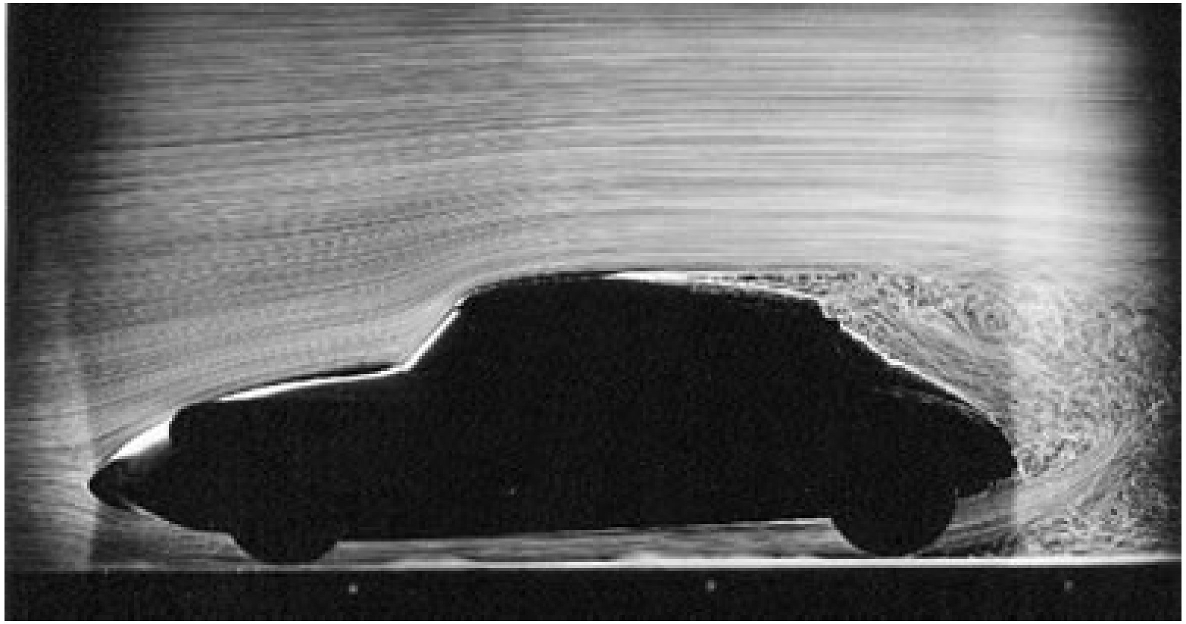
Source: Terry Goss, Guadalupe 2006

Example for incompressible flow (2)

Flow around a U-Boat

Source: National Oceanic and Atmospheric Administration

Example for incompressible flow (3)



Airflow with M < 0,3 (equals *v* < 370 km/h)

Source: Onera

Example for incompressible flow (4)

Flow around a glider

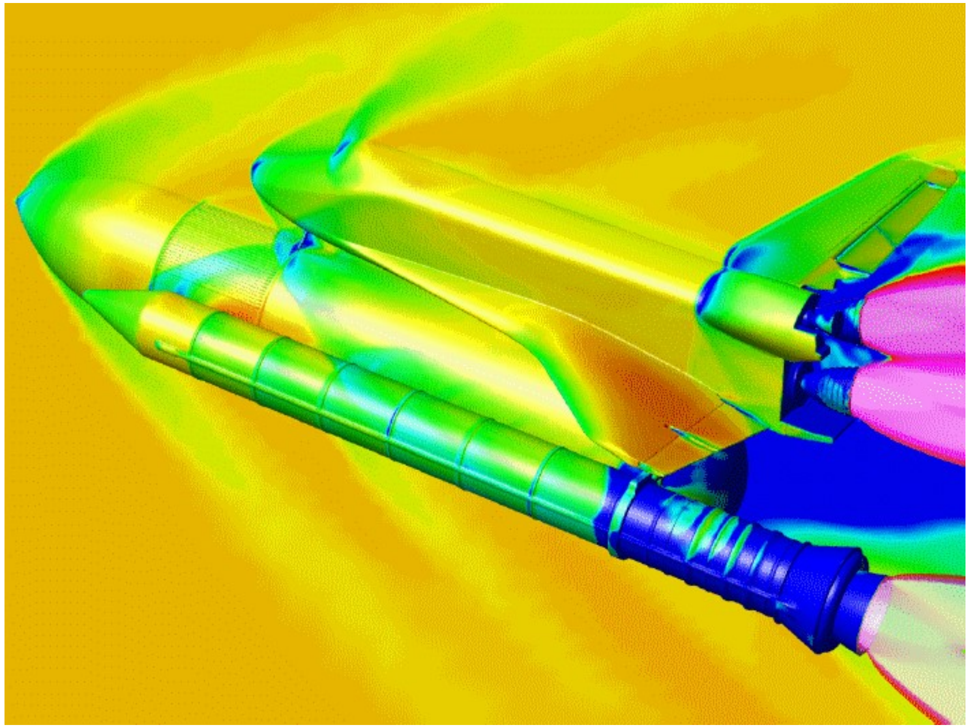
Source: Akaflieg Universität Stuttgart

Compressible flow

Once the density variation within a flow field cannot be neglected, the flow needs to be considered compressible.

One significant feature of compressible flows is the appearance of discontinuity (pressure shocks).

Examples for compressible flow (1)

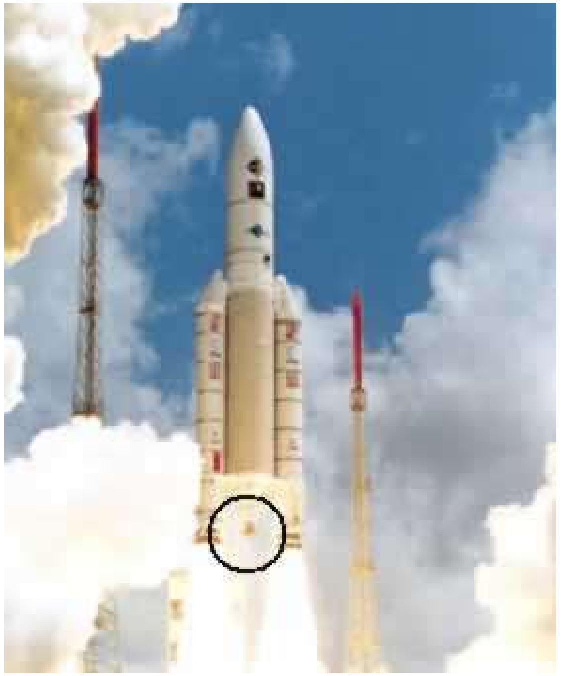


Source: Ansys

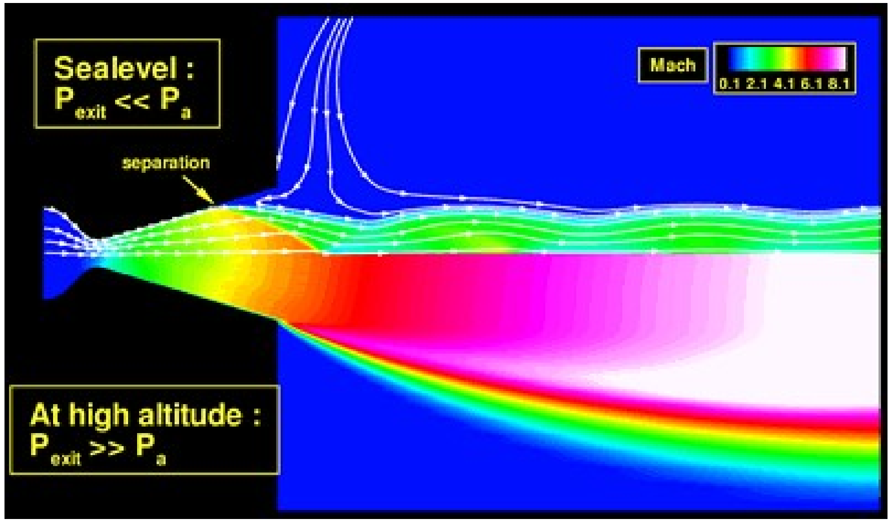
Source: NASA

- Very high velocities
- Compaction shock at the tip

Examples for compressible flow (2)



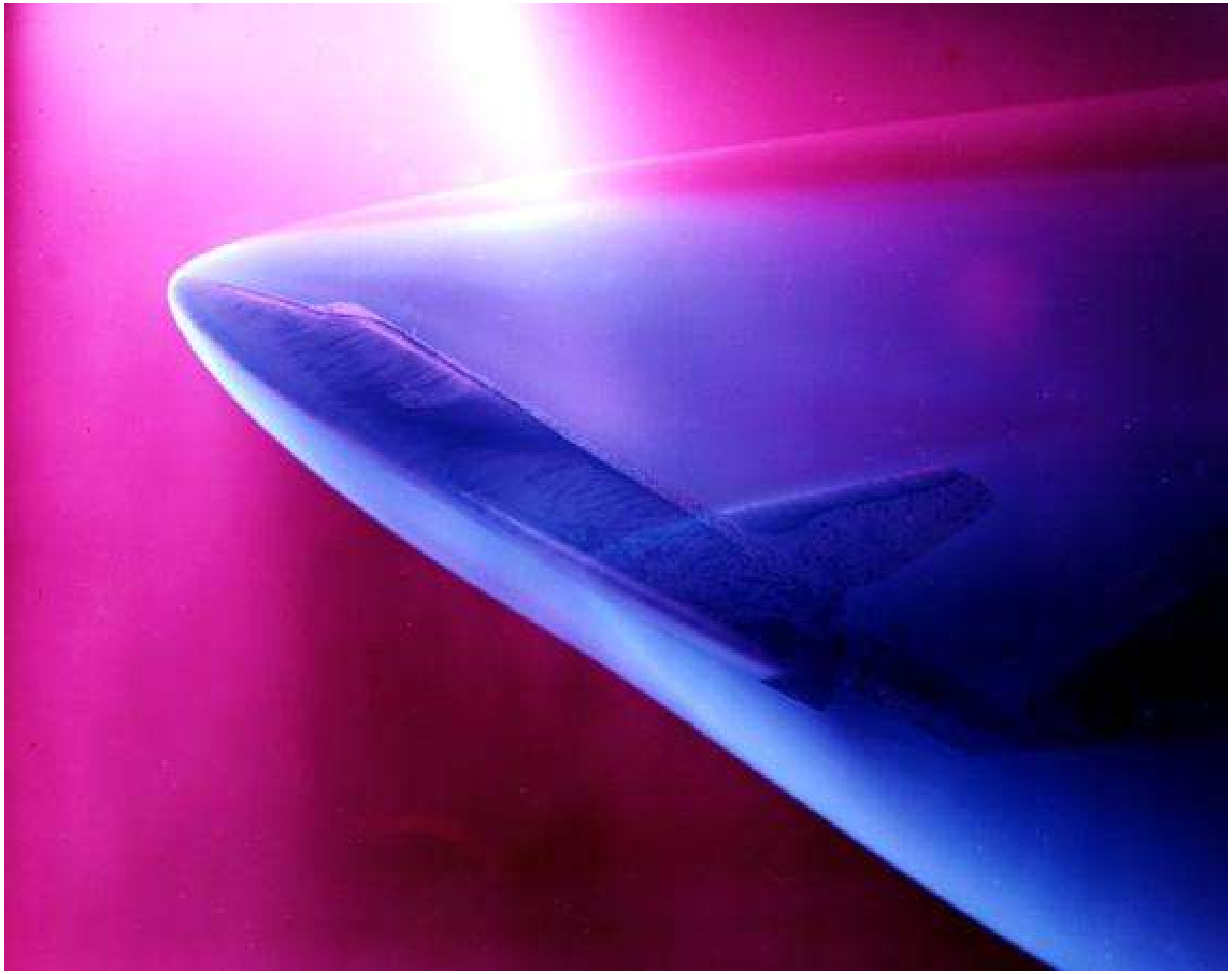
Source: Onera



Source: Onera

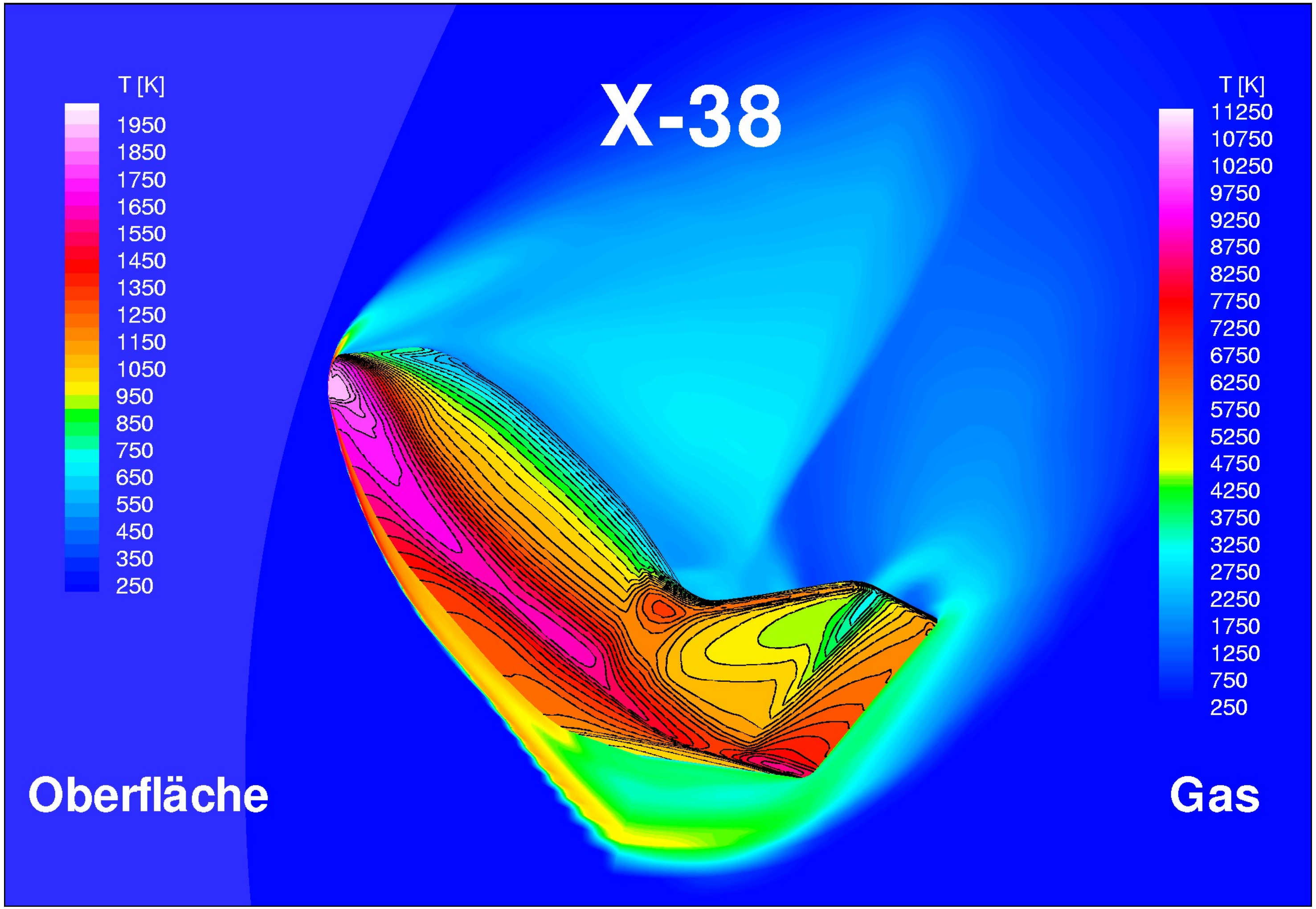
- Acceleration of a fluid within a nozzle

Examples for compressible flow (3)



Quelle: NASA

Examples for compressible flow (4)



Quelle: IRS Universität Stuttgart

Examples for compressible flow (5)

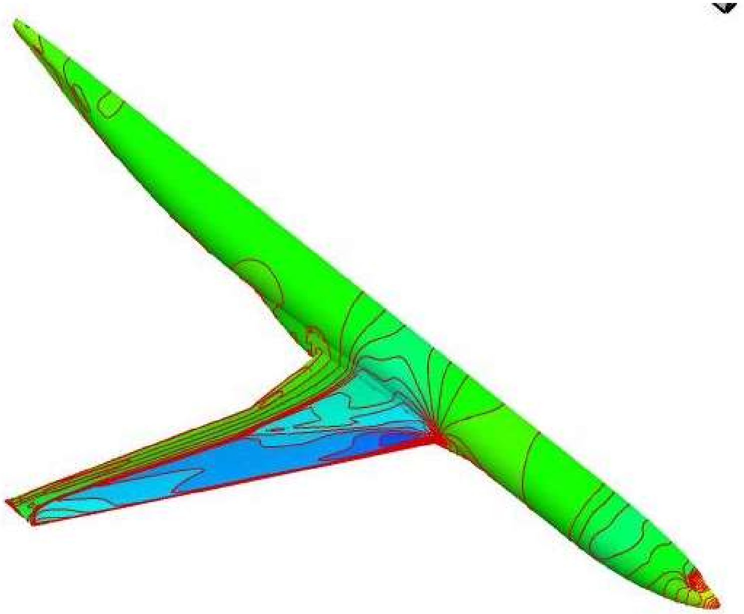
Quelle: US Navy

Examples for compressible flow (5)

Quelle: US Navy

Examples for compressible flow (6)

Quelle: DLR



Quelle: IAG Universität Stuttgart

- Aircraft at transonic speed

Examples for compressible flow (7)

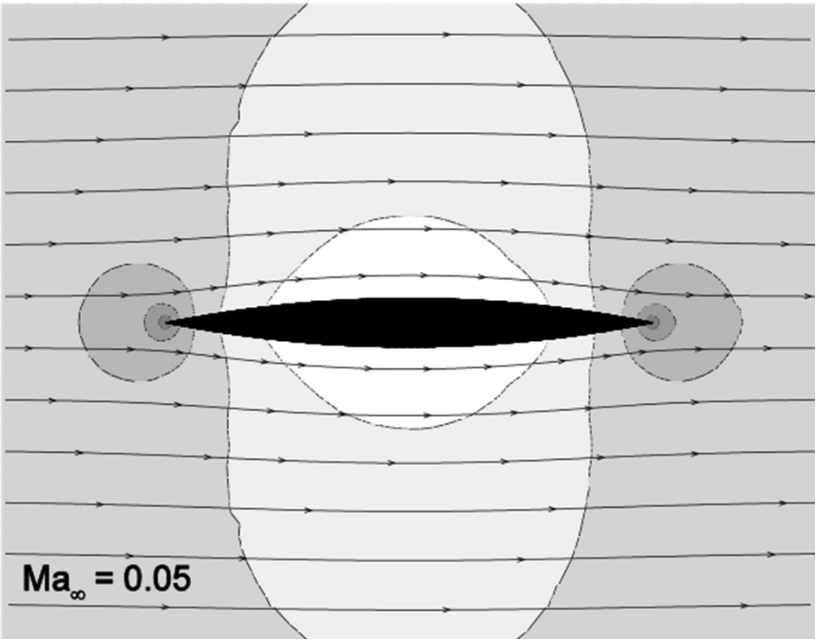
Examples for compressible flow (8)

From the incompressible to the compressible regime

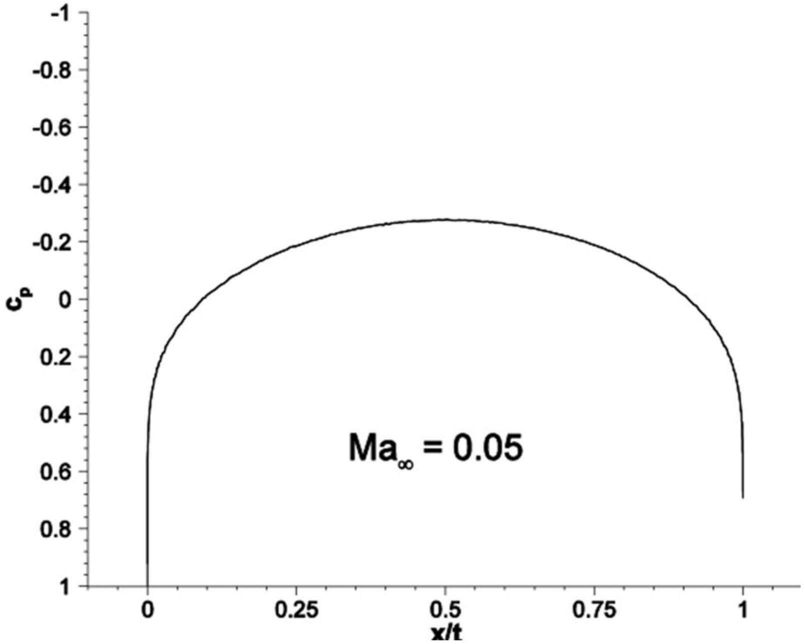
Using an airfoil with different Mach numbers the changes in the fluid flow are explained.

Airfoil at M = 0.05

Mach number

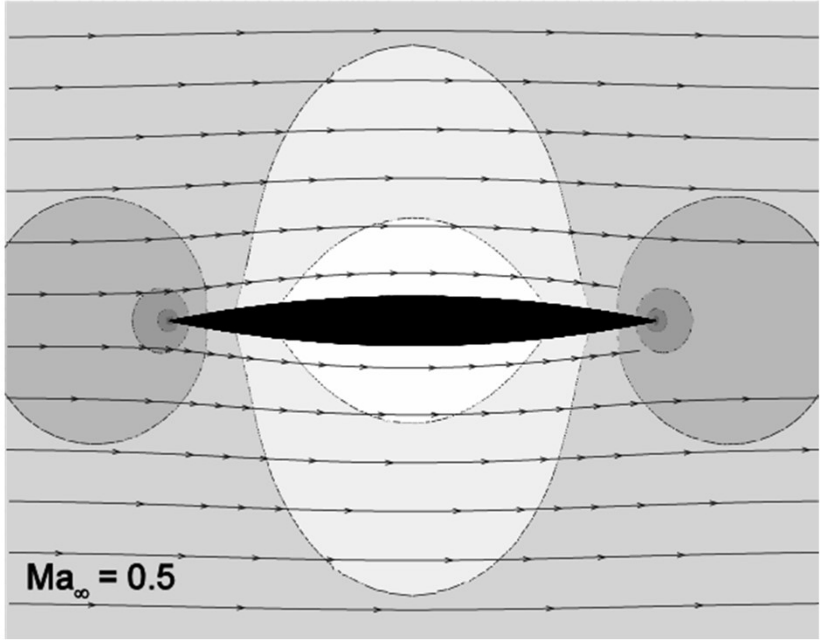


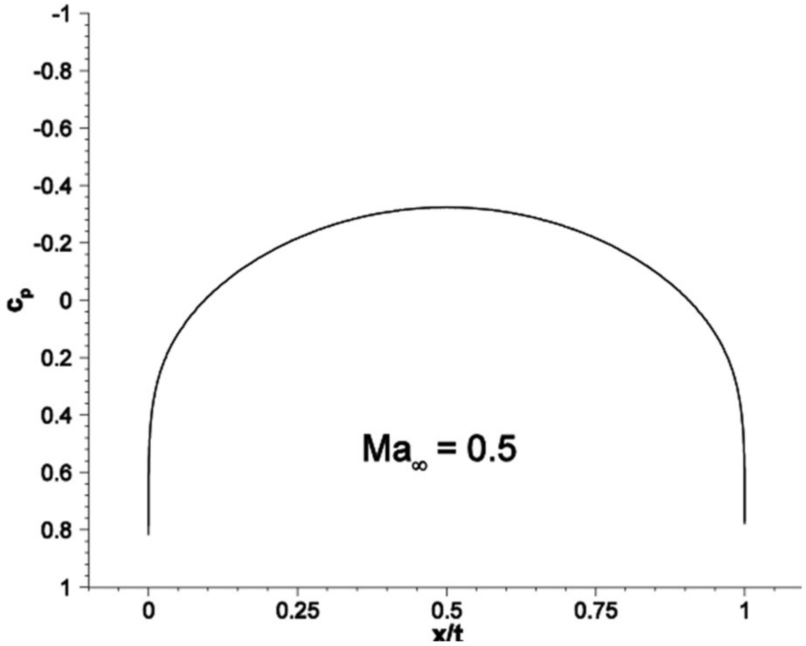
Pressure distribution



- Symmetric pressure distribution

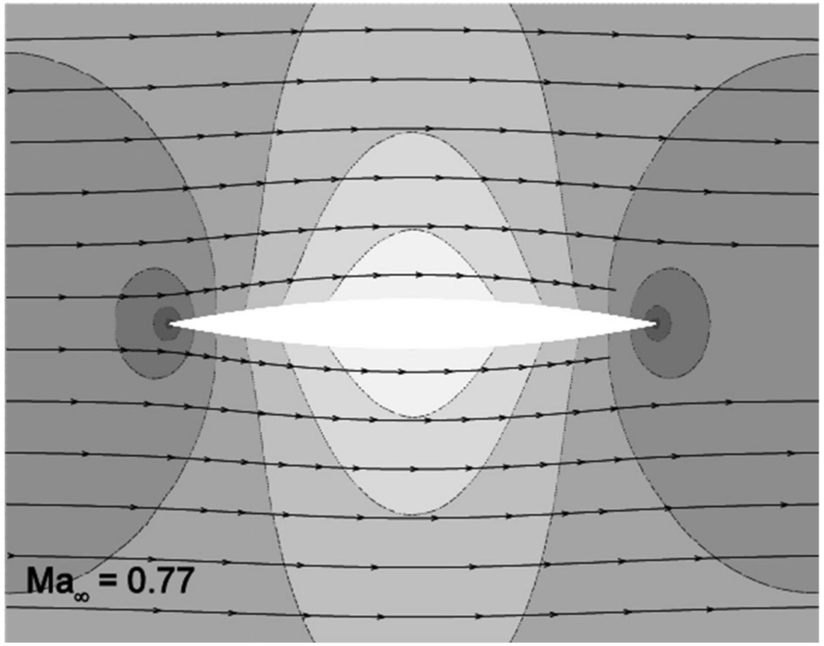
Airfoil at M = 0.5

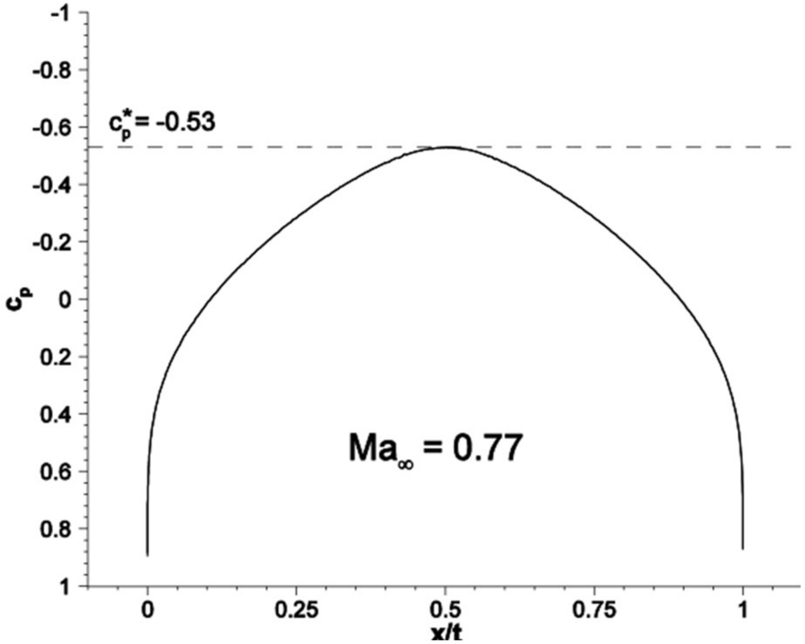




- Symmetric pressure distribution

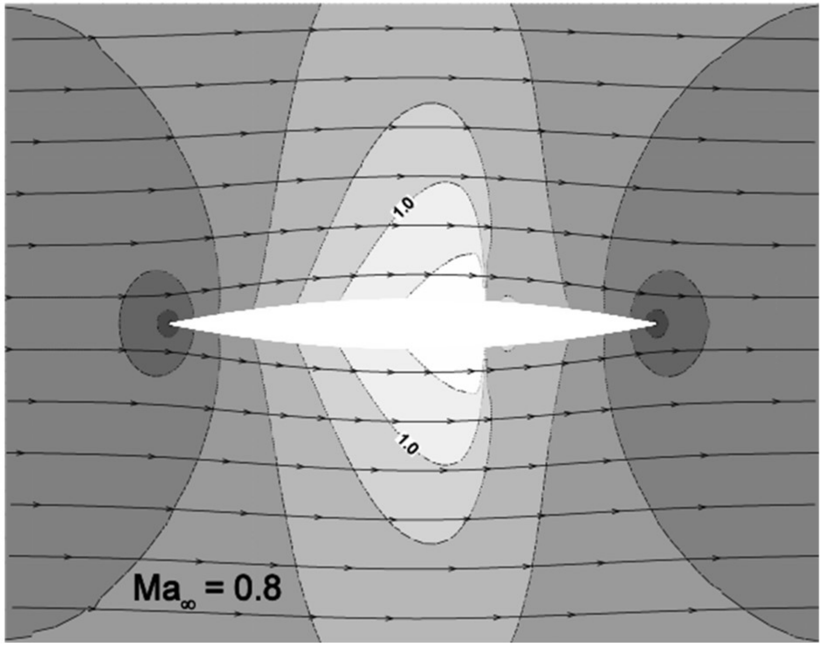
Airfoil at M = 0.77

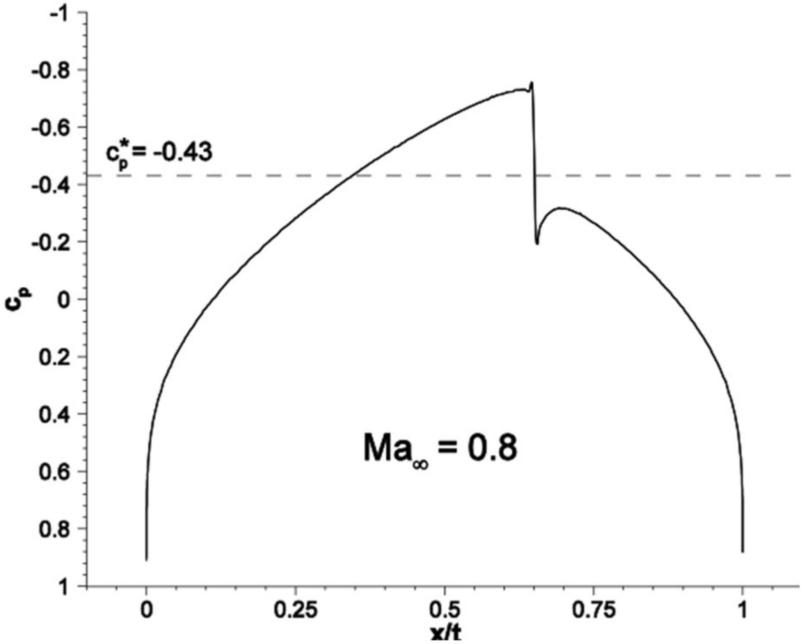




- At the airfoil top the speed of sound is barely reached

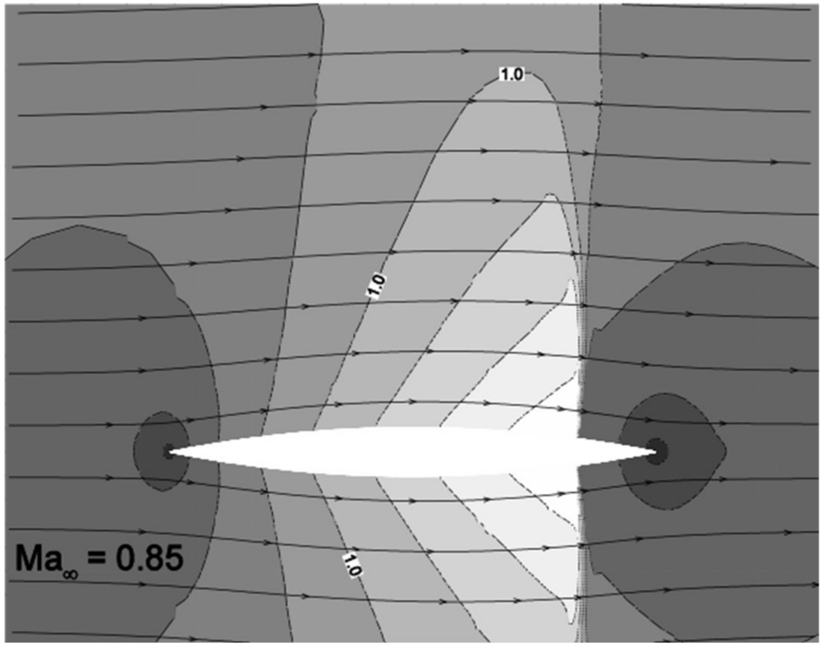
Airfoil at M = 0.8

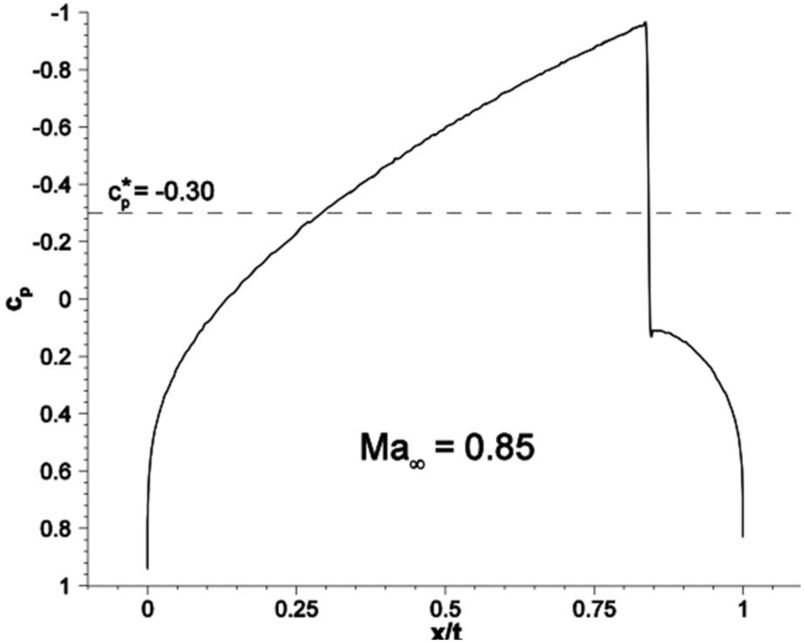




- Formation of a local supersonic region, which is closed by a shock

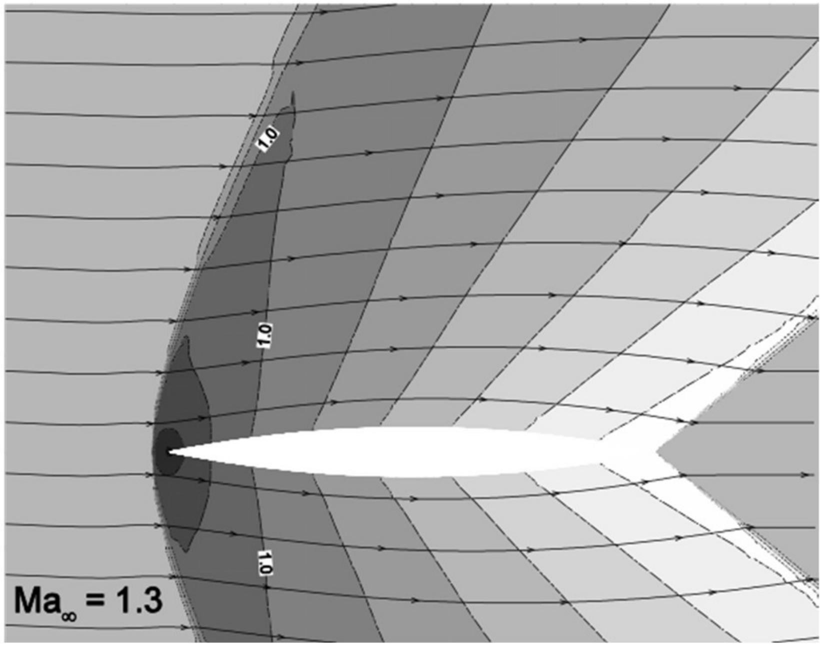
Airfoil at M = 0.85

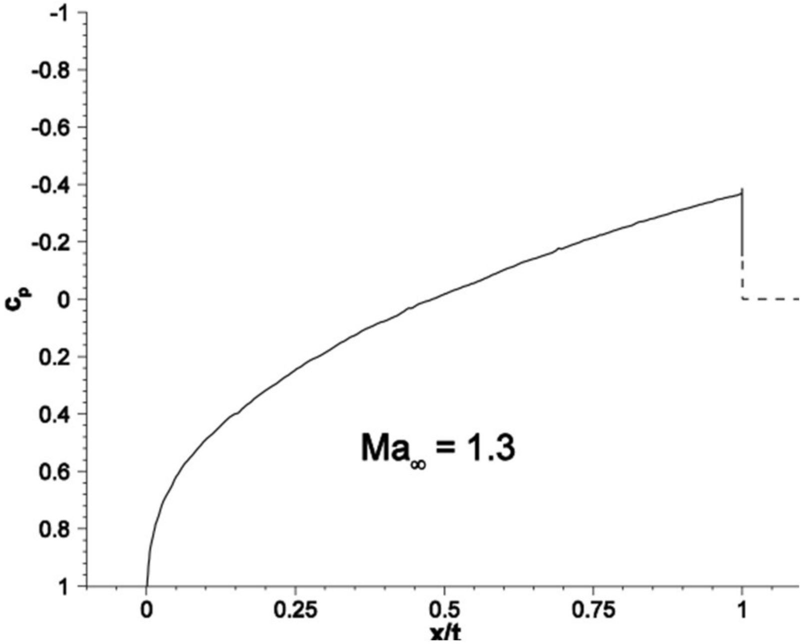




- Enlargement of the supersonic region; increase in shock intensity

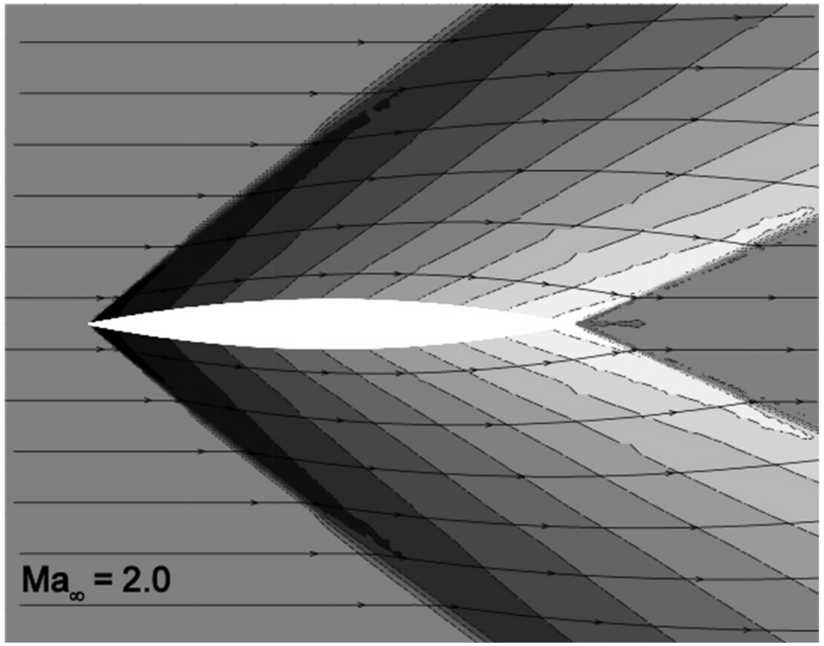
Airfoil at M = 1.15

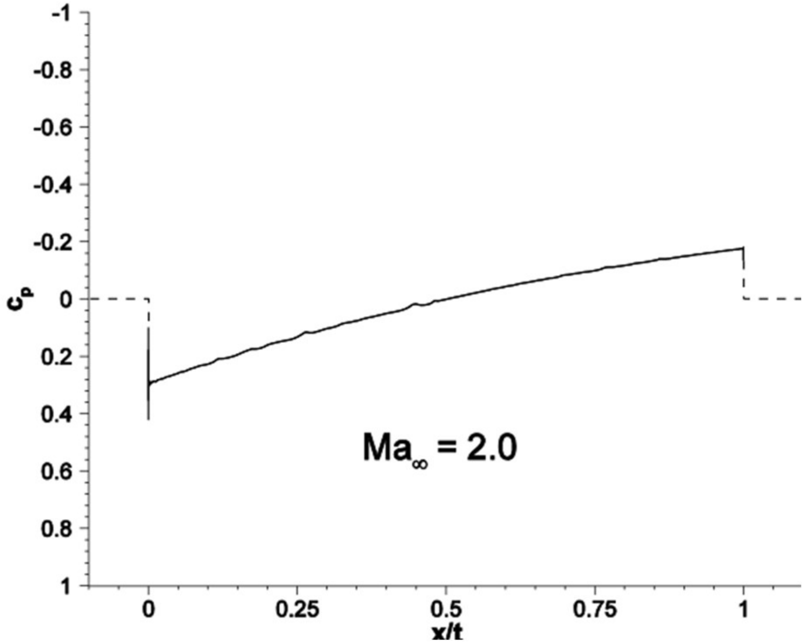




- Supersonic inflow leads to a detached head wave (bow shock) – region with high losses
- Disturbances travel only downstream

Airfoil at M = 2





- Attached shock at tip

Flow with and without friction

For some cases it is justifiable to neglect the friction. The Euler equations are then used.

An important parameter for the interaction between friction and velocity is the Reynolds number:

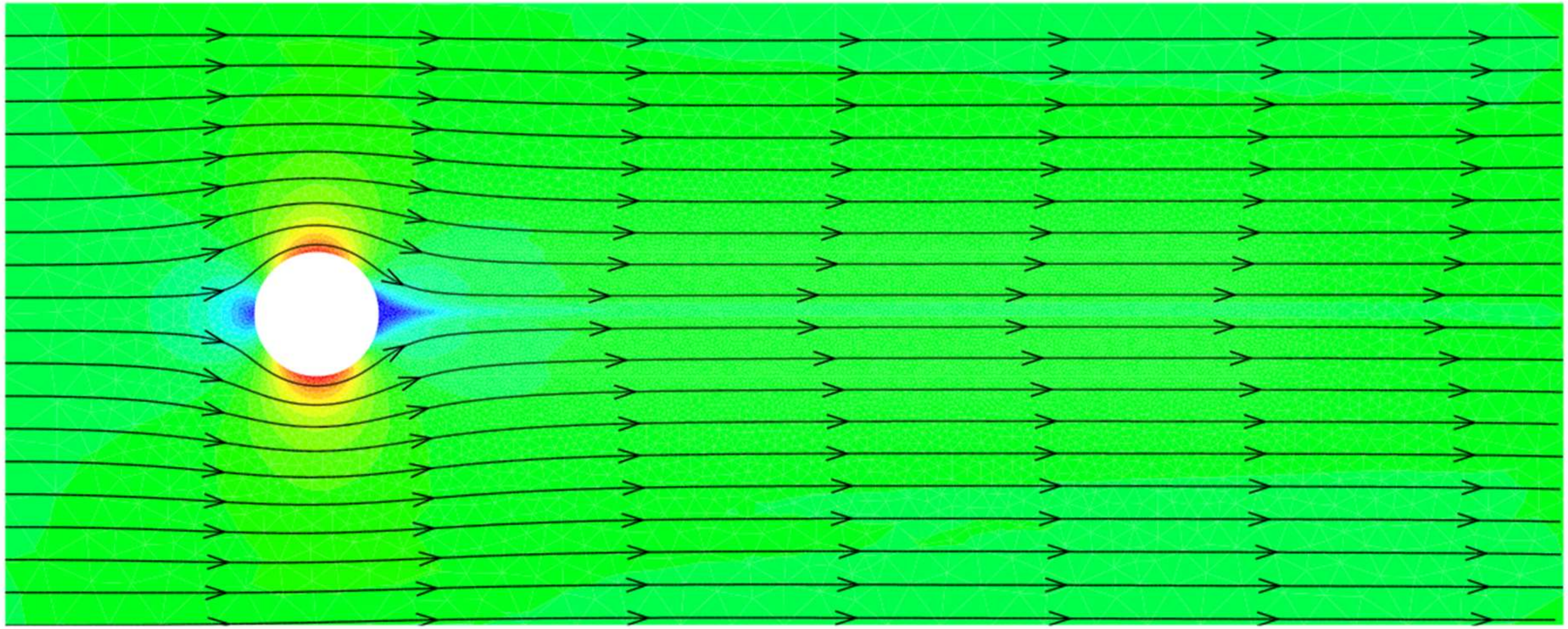
$$\mathrm{Re} = \frac{\rho \cdot x \cdot v}{\mu}$$

The Reynolds number is an important similarity parameter in fluid mechanics.

Flows with equal Reynolds number behave the same:
Flow around a body x = 10 m, v = 10 m/s results in the same as
the flow around x = 1 m, v = 100 m/s, using the same density and viscosity.

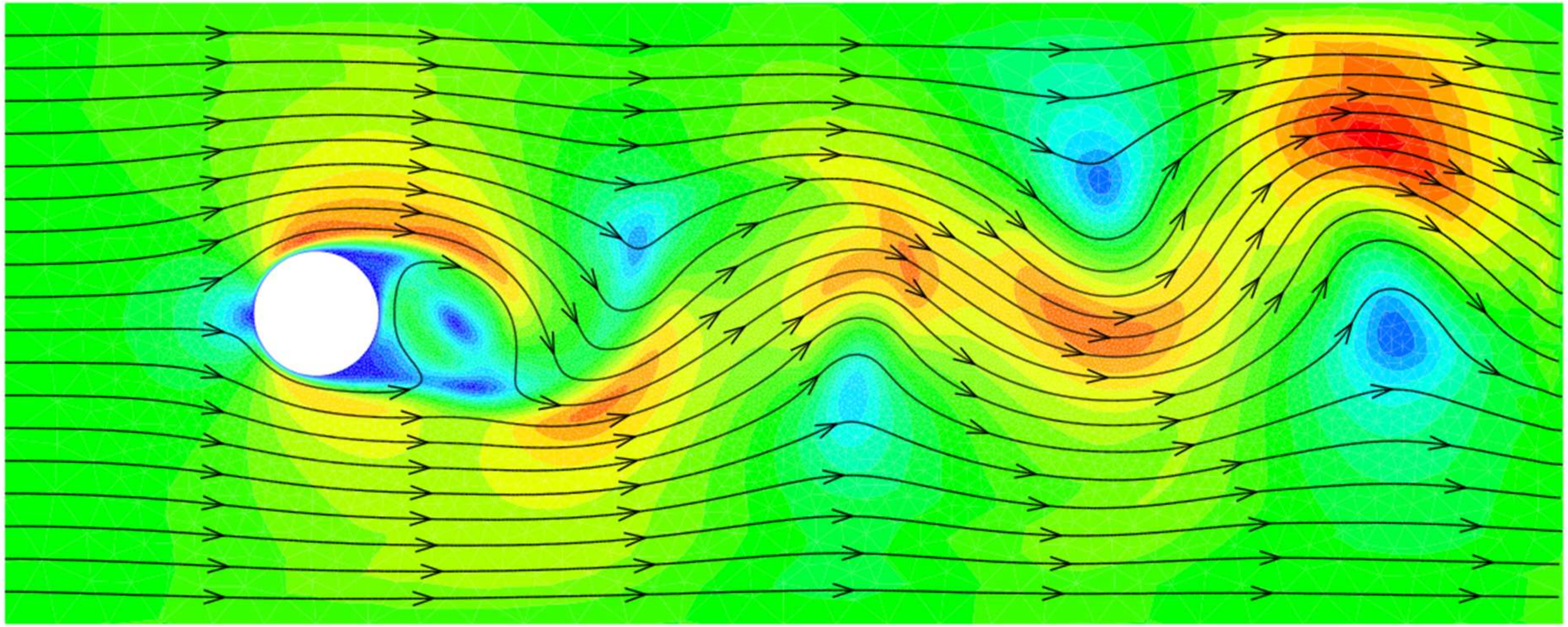
→ Important for measurements in wind tunnel.

Flow around a cylinder – Euler equations



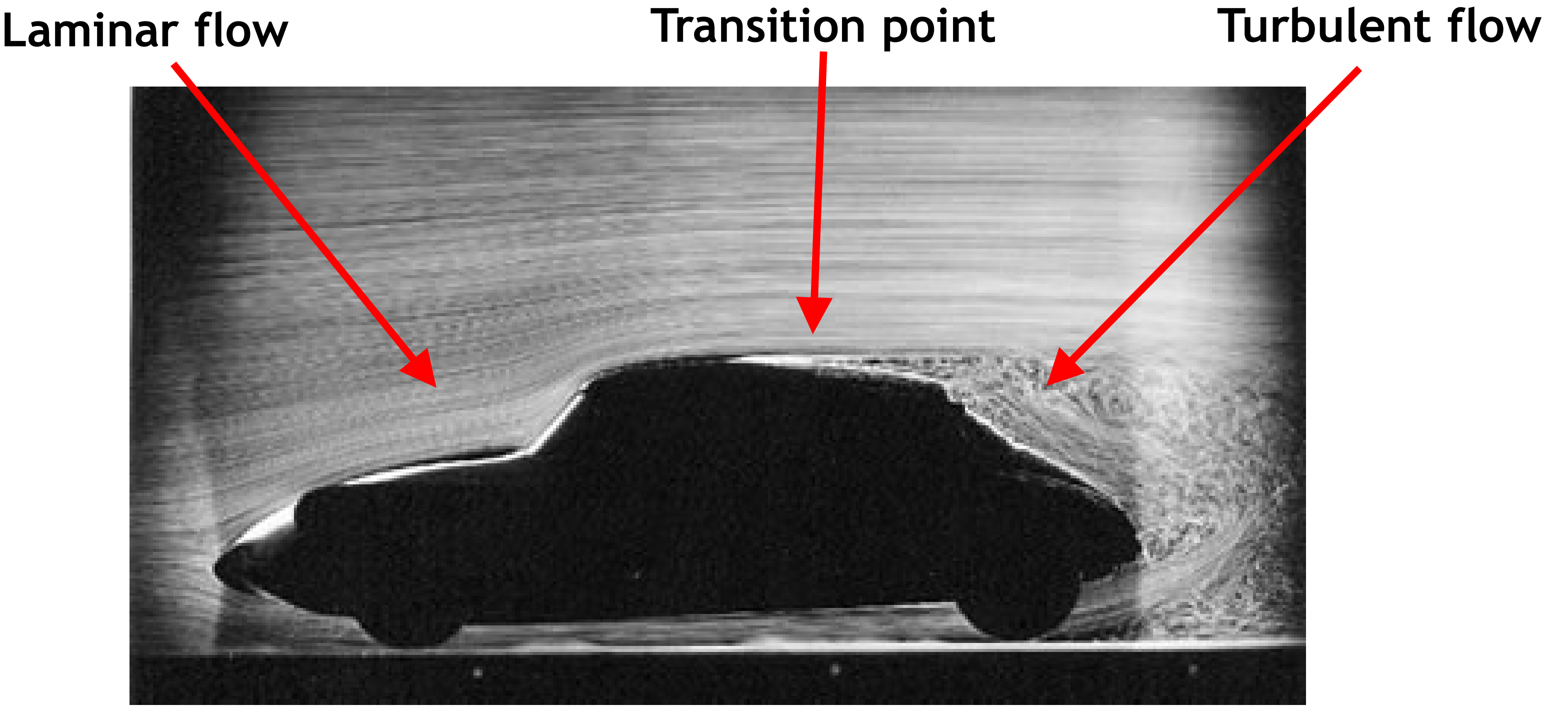
- Perfectly symmetrical

Flow around a cylinder - Navier-Stokes equations (Reynolds number small)



- Formation of a boundary layer as well as a vortex street (non-stationary flow)
- Using Euler equations at small Reynolds numbers results in the same flow field, which is wrong.

Laminar and turbulent flow



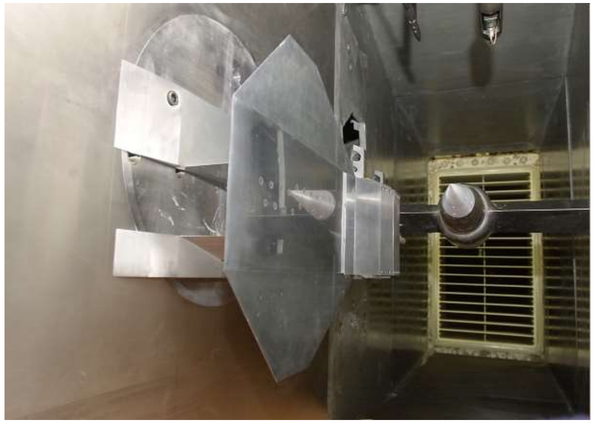
Turbulent flow (almost always the case) is one of the biggest challenges for todays CFD-methods. The small turbulent scales can usually not be resolved but have an impact on the flow field.

Important phenomena of compressible flow

The numerical methods that are discussed in this course are based on the numerical treatment of the basic phenomena of compressible flow.

The experimental facility to explore these phenomena is called a shock tube.

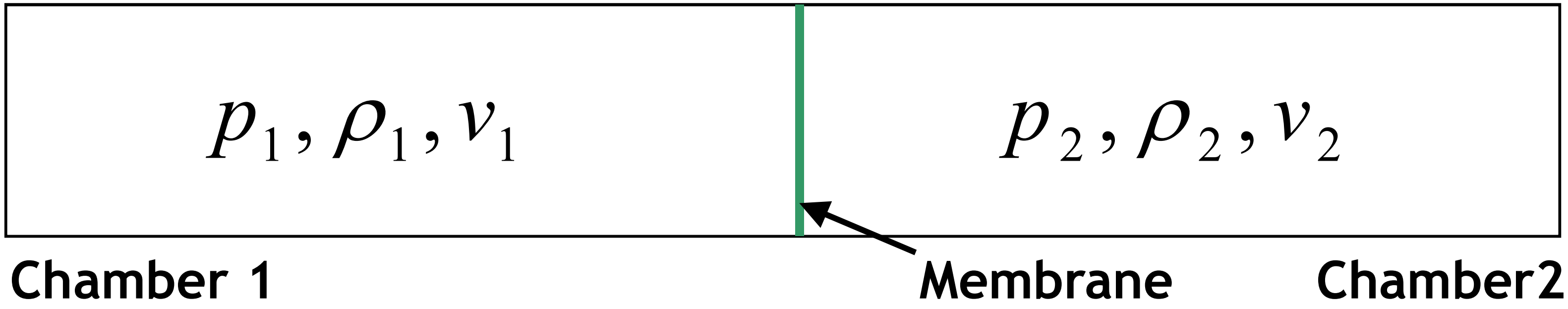
Das Stoßrohr



Quelle: IAG Universität Stuttgart

Shock tube

Used for the experimental investigation of gas-dynamic phenomena.
How it works: A long tube is split into two chambers by a membrane.
Chamber 1: High pressure, Chamber 2: Reduced pressure



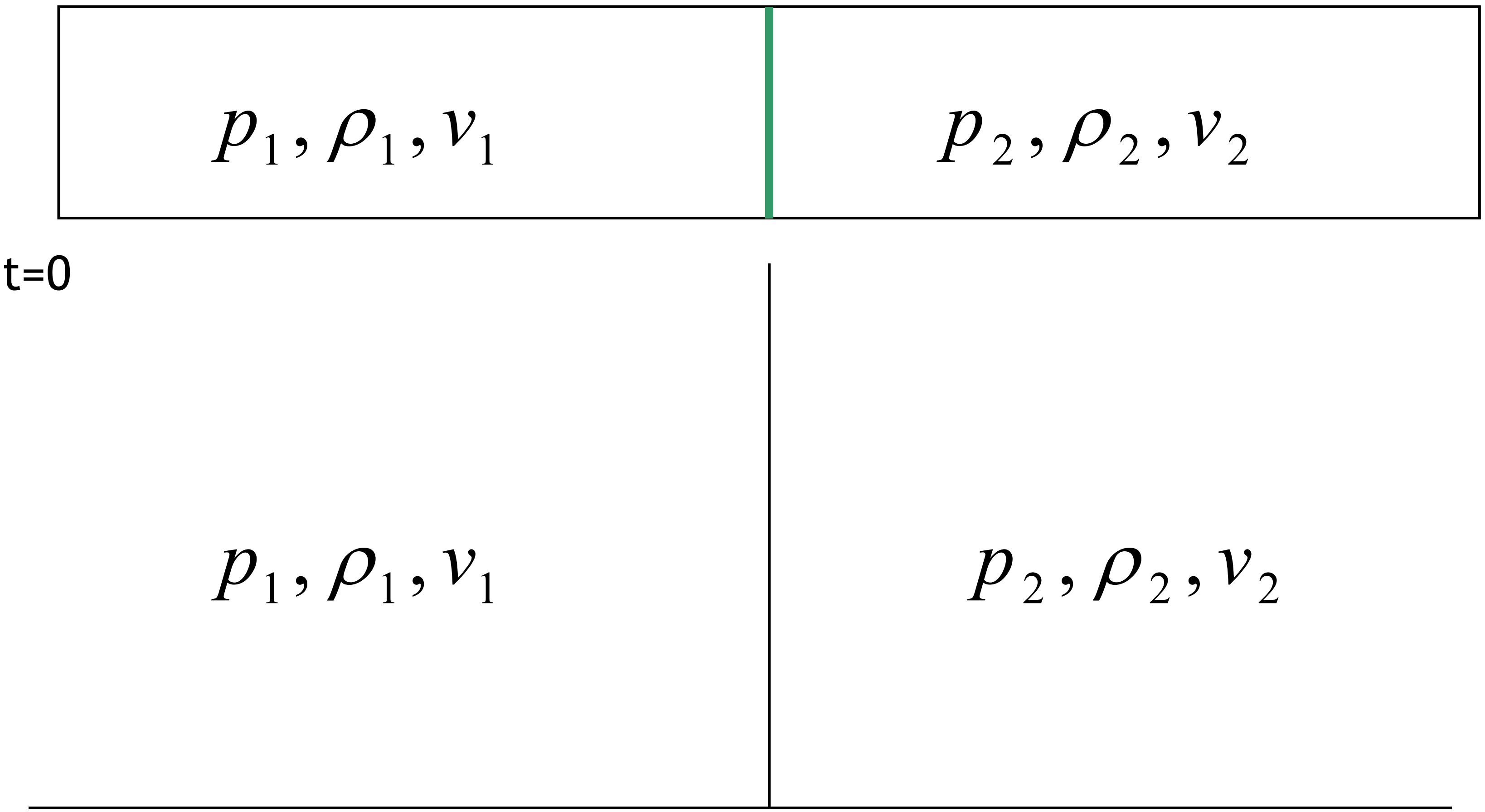
p_1, ρ_1, v_1 p_2, ρ_2, v_2

Chamber 1 **Membrane** **Chamber2**

At the beginning of the experiment the membrane is destroyed.

As a result, a pressure compensation process is started, with which the gas can be accelerated to supersonic speed. Hereby different gas dynamical phenomena can be observed.

Evolution of the phenomena

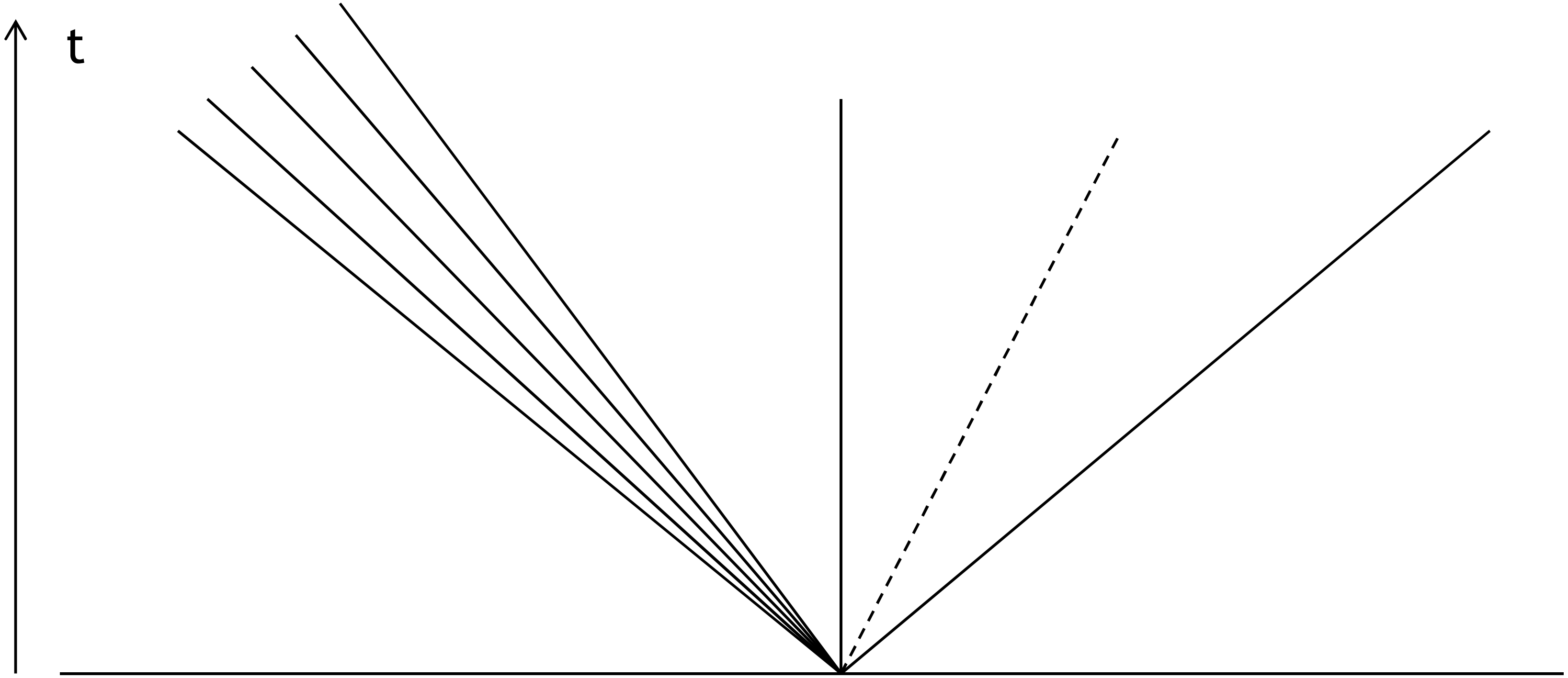


p_1, ρ_1, v_1 p_2, ρ_2, v_2

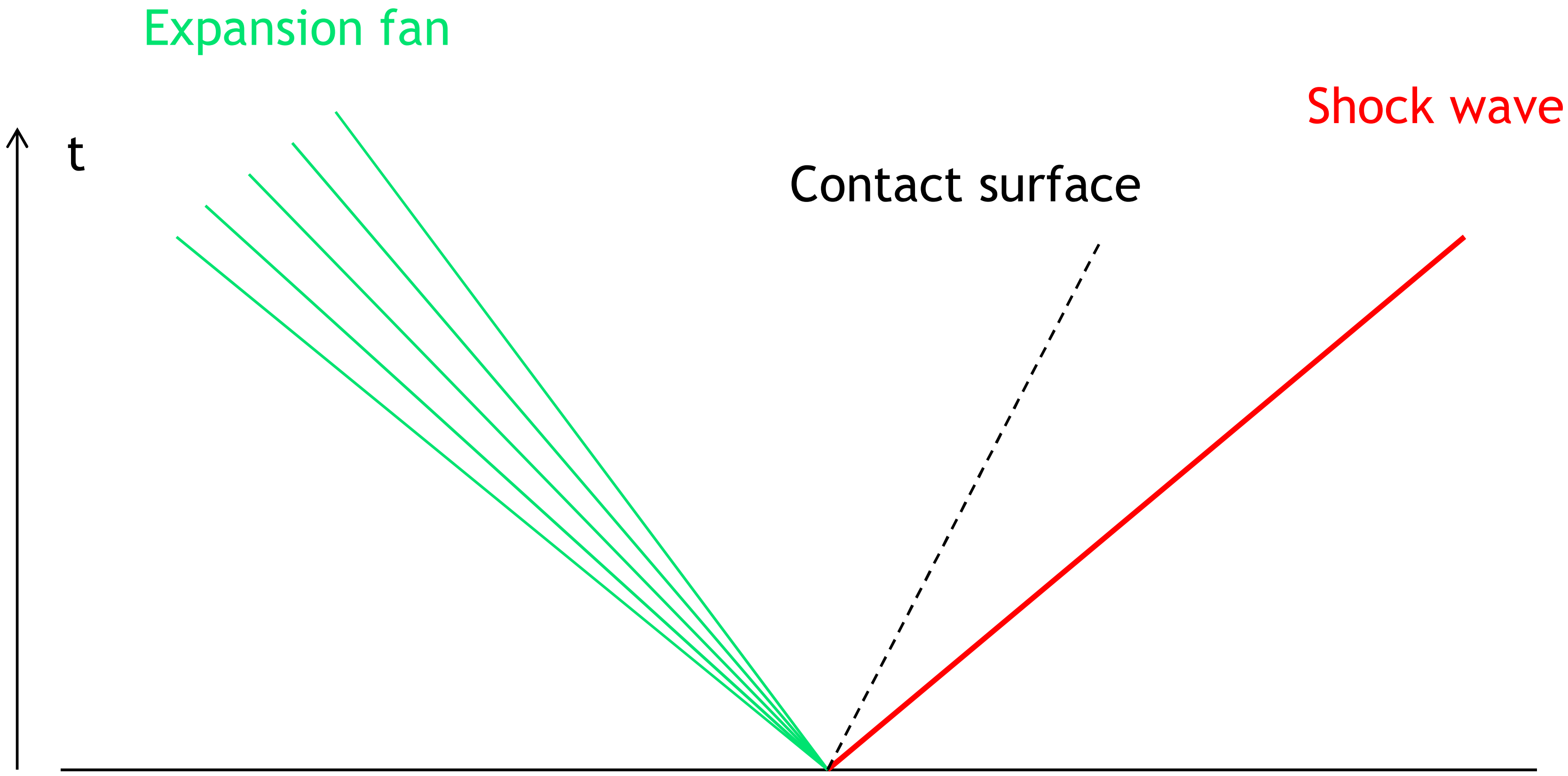
t=0

p_1, ρ_1, v_1 p_2, ρ_2, v_2

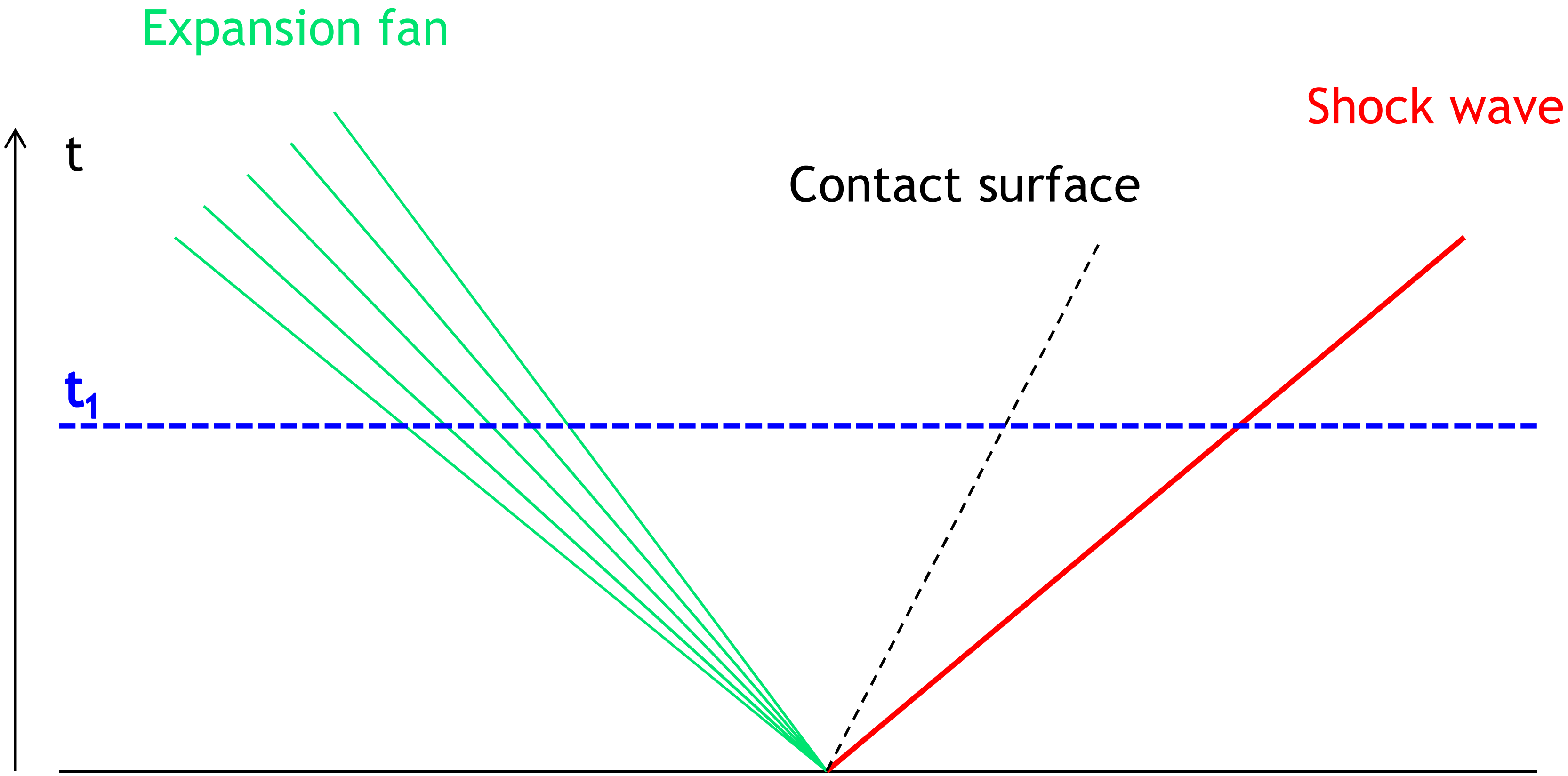
Evolution of the phenomena



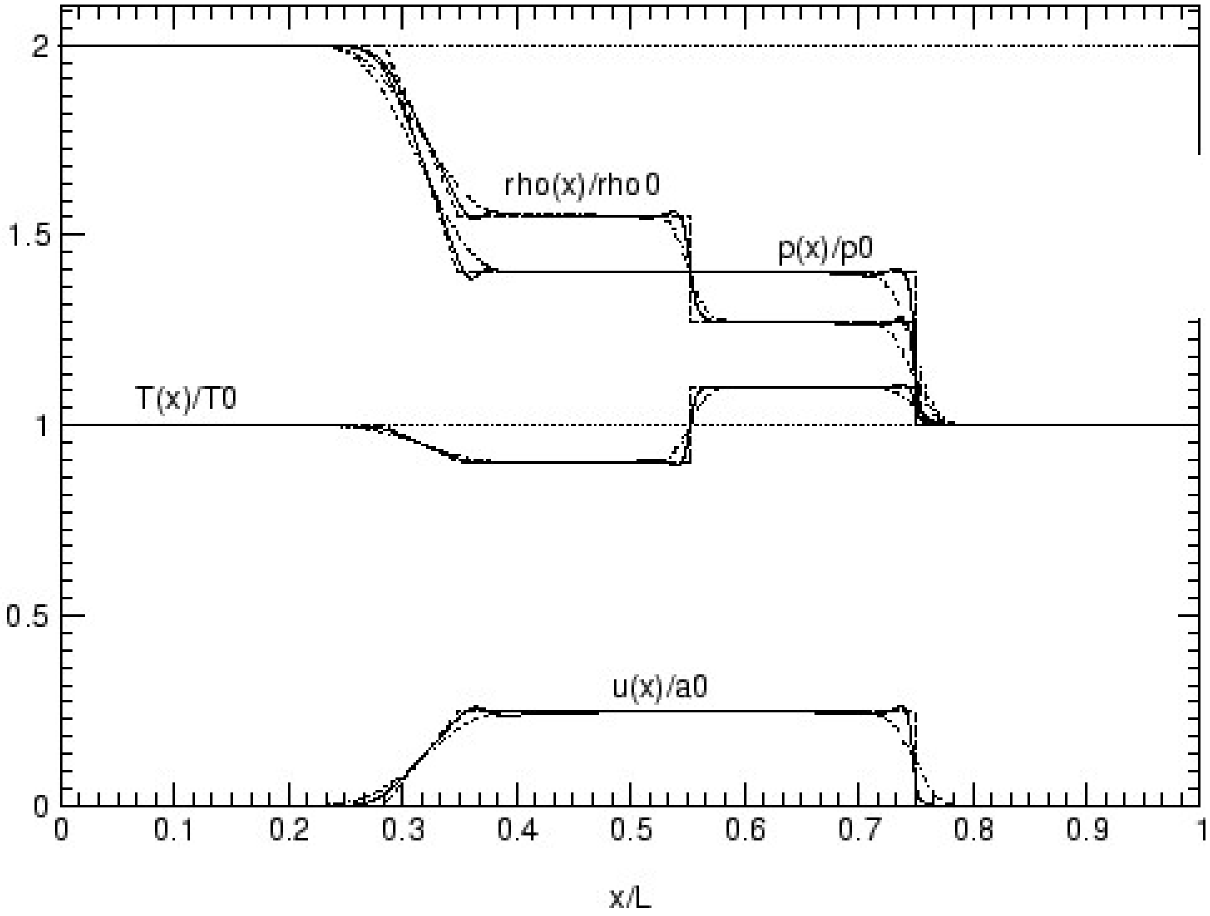
Evolution of the phenomena



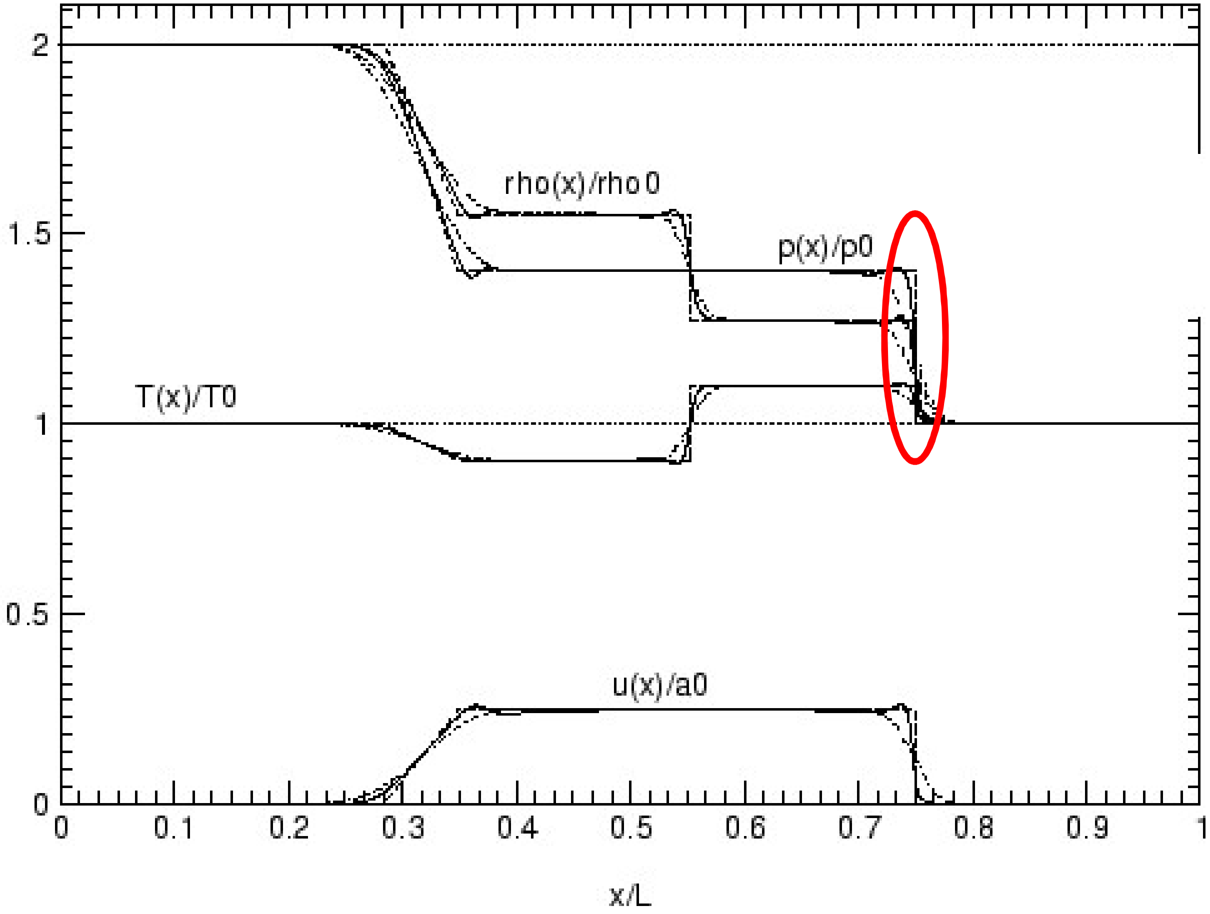
Evolution of the phenomena



Shock tube (2)

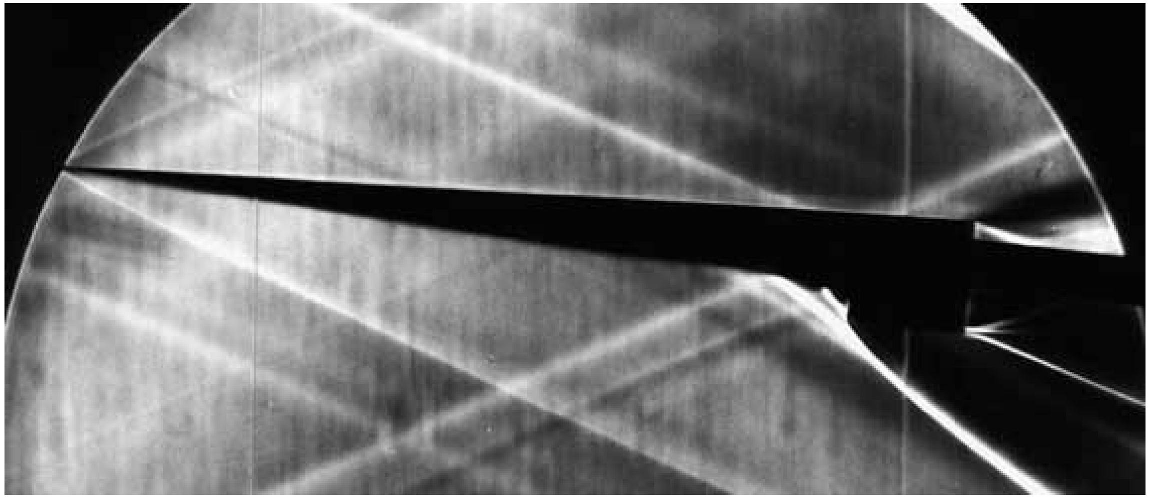


Phenomena within a shock tube - shock



Phenomena within a shock tube - shock

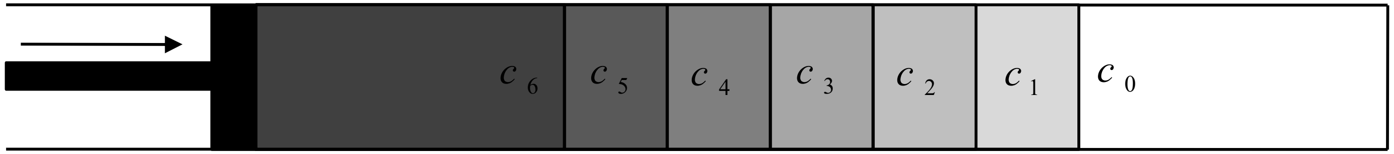
- The (compression) shock is a gas dynamic phenomenon, whereby changes of density, pressure, velocity and all connected parameters happen in a very small spatial scope.
- A shock is always related to an increase of entropy. It leads to losses in the total pressure, which are irreversible.
- Example: Supersonic boom.



Shock system around a test-body at Mach 2.5

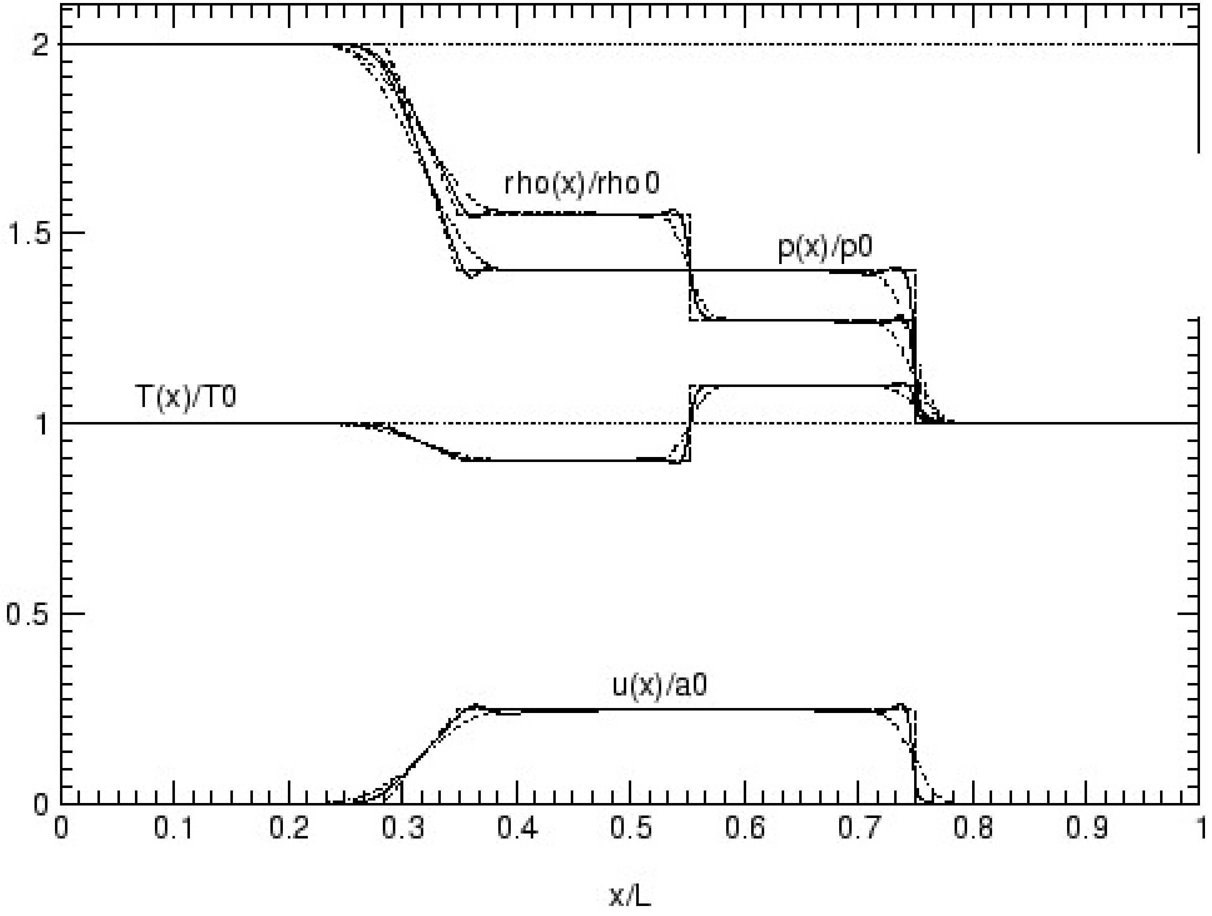
The creation of a shock

- Model: Cylinder, in which a piston is moved abruptly several times
- Each change in piston velocity creates a disturbance, which moves at the speed of sound
- Each disturbance entails small changes in the state parameters, i.e. the speed of sound is higher behind the piston before

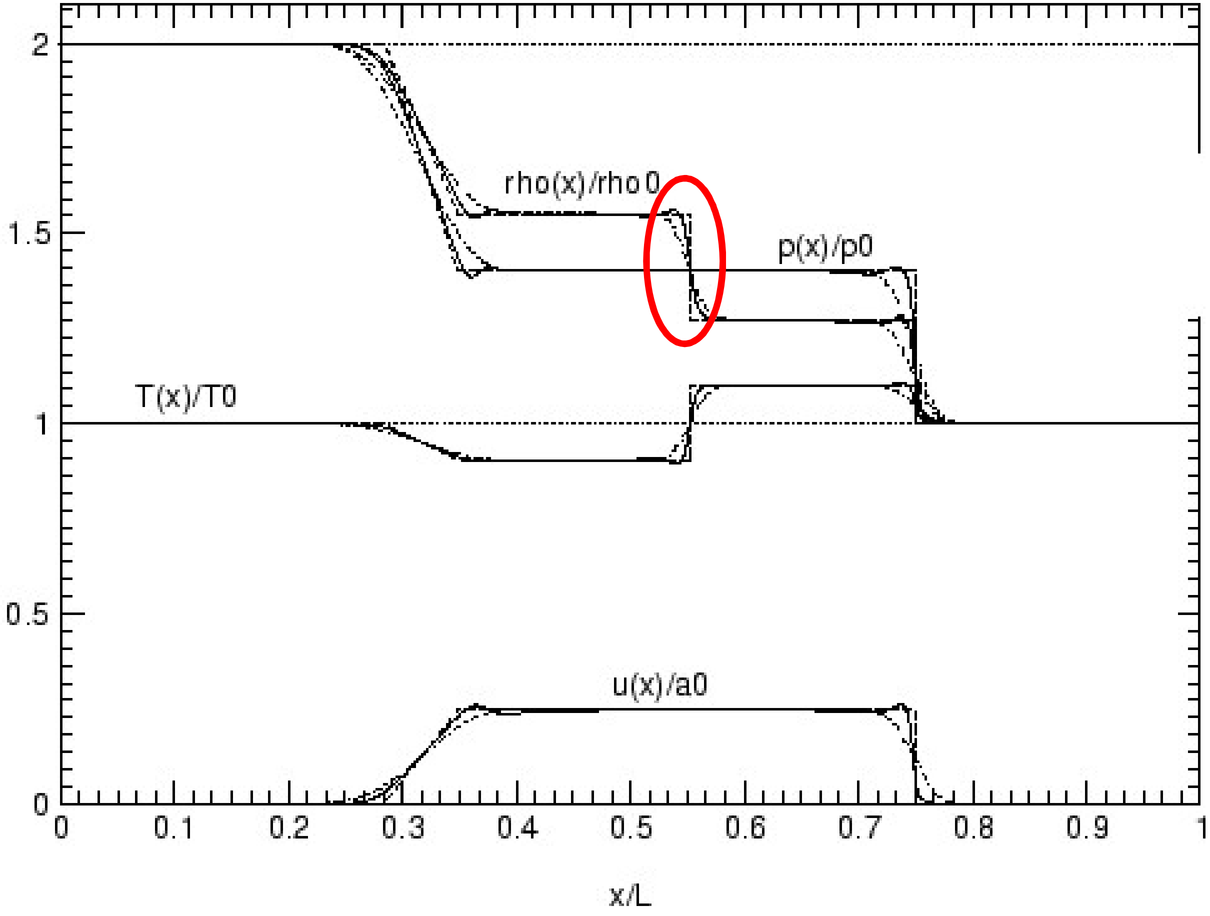


- The first disturbance is the slowest, the last one the fastest. Therefore the disturbances catch up to another.

Phenomena within a shock tube - contact surface



Phenomena within a shock tube - contact surface



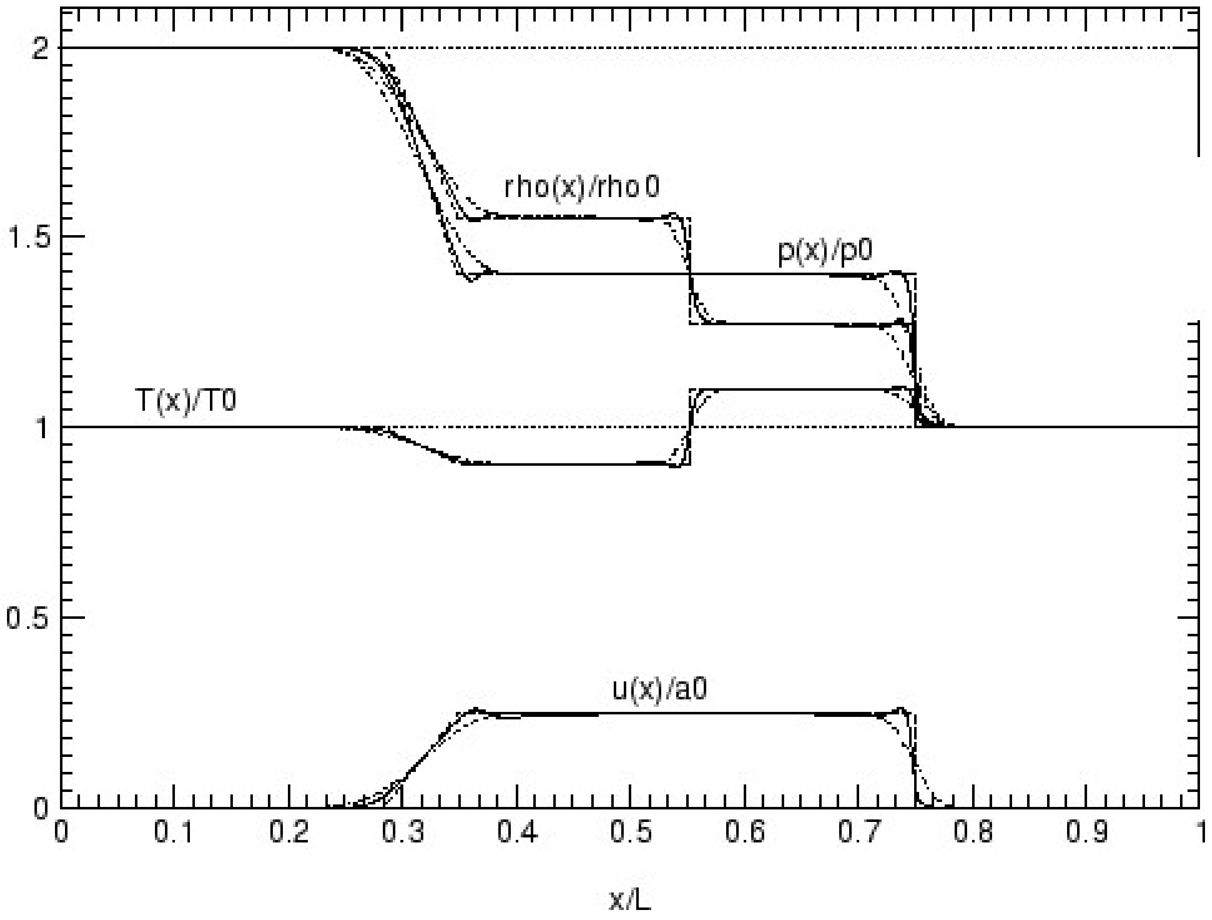
Phenomena within a shock tube - contact surface

- In fluid mechanics a contact surface represents a jump in density or the change of material.

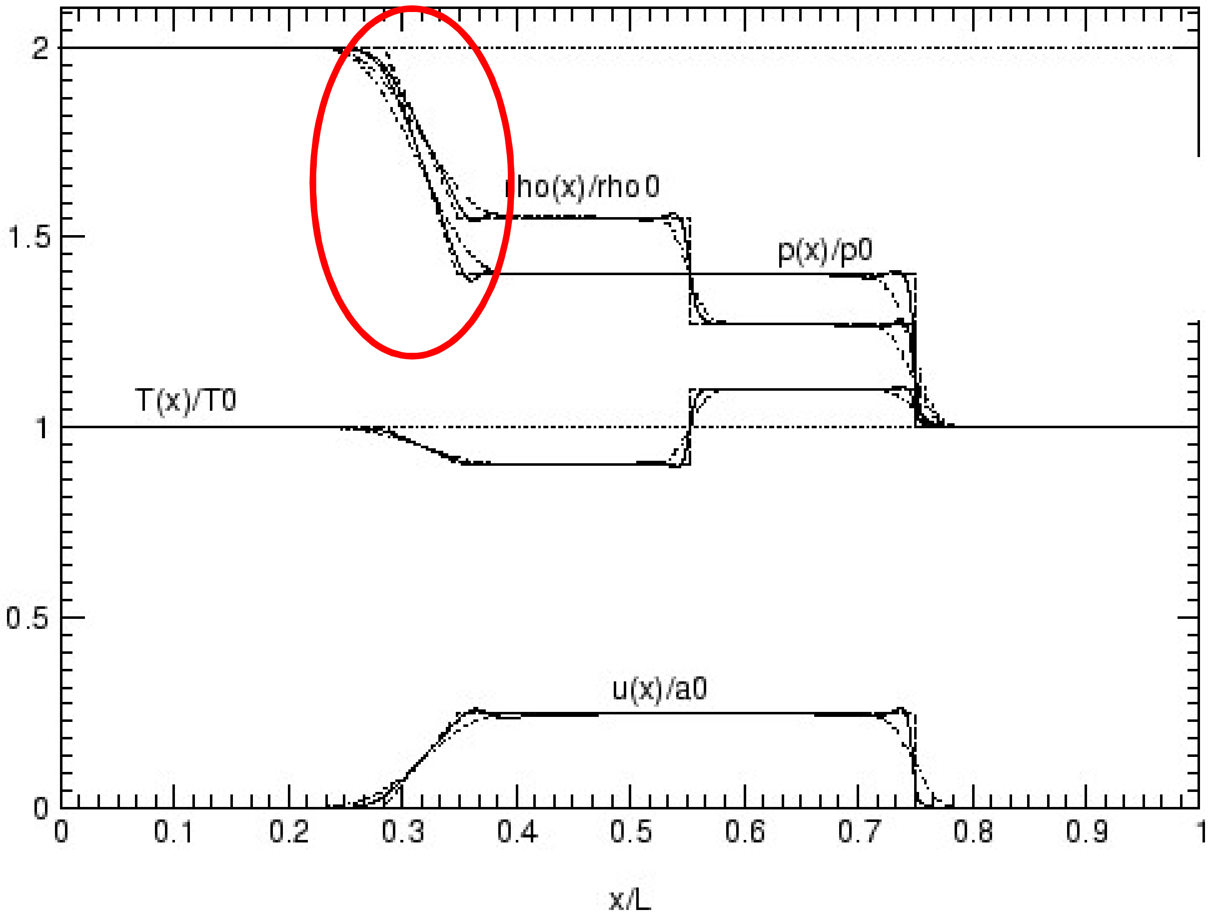
- If there is only a change in density while pressure and velocity are equal, there is no change in state, when using the Euler equations (frictionless).

- The contact surface within the shock tube is created because the pressure is faster equalized than the density. Once the pressure difference is gone, there is no driving force anymore, that could equalize the density.

Phenomena within a shock tube - expansion fan



Phenomena within a shock tube - expansion fan



Phenomena within a shock tube - expansion fan

- In comparison to the shock, the behavior of the expansion fan is quite the opposite. The fluid is getting thinner as it flows downstream.

- A jump in values similarly to the shock is not possible. It would entail, that the entropy over the „expansion shock“ would sink, which is physically impossible. Therefore it is a continuous change that is also isentropic.