Simulation of turbulent flows

HPCFD07

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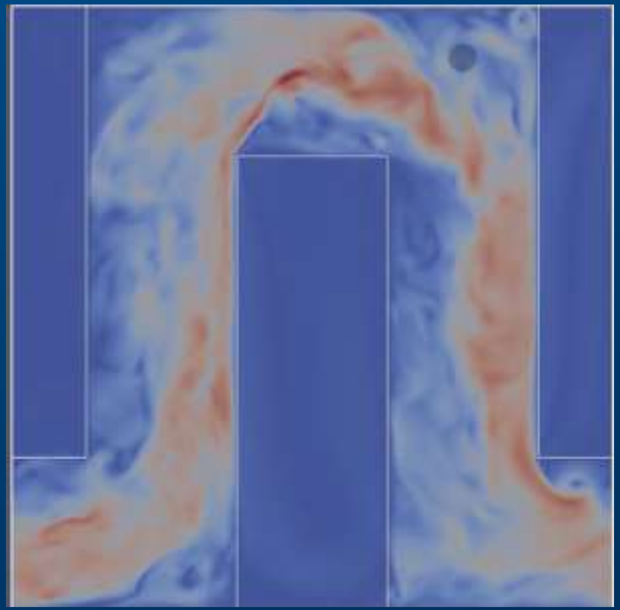

[Hafemann, Fröhlich, ISM]

Course „Introduction to CFD“, HLRS, Dresden, 5.-9.2.2024

Motivation

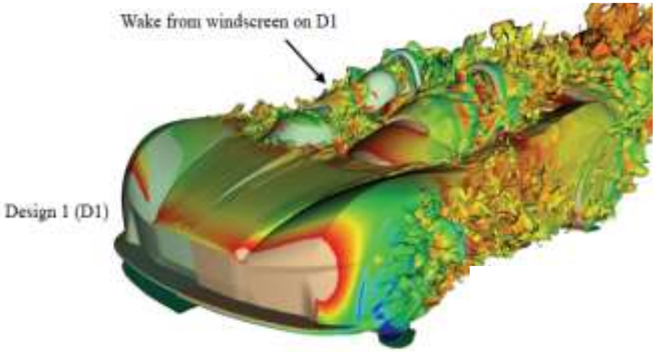
- Environmental flows
  - Weather
  - Rivers, lakes
  - Pollution
- Flows around objects
  - Cars
  - Planes
  - Trains
  - Buildings
  - Sports
- Internal flows
  - Pipes, ducts, valves
  - Combustion devices

[www.uni-koeln.de]

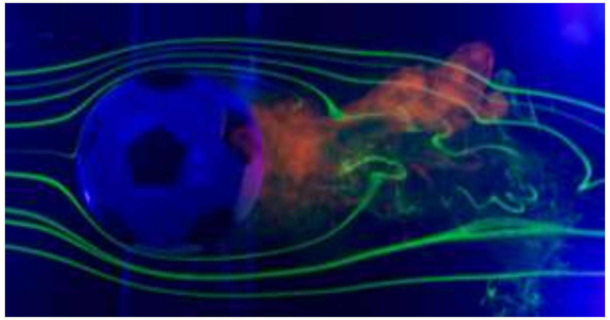
[Armin Weigel/dpa]

Water surface Elbe

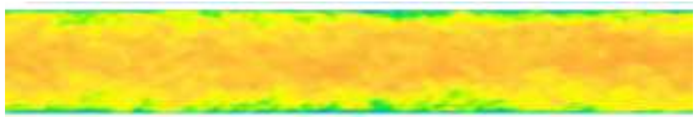
[Fröhlich]



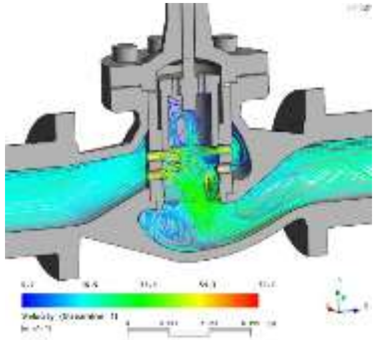
[Mengaldo et al. 2021]



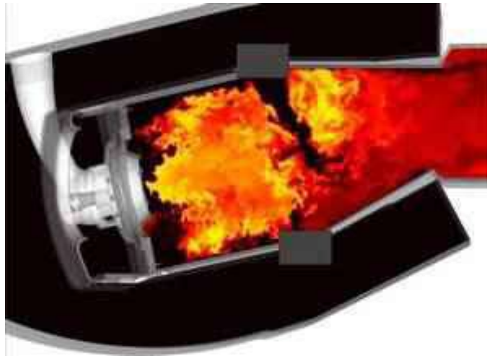
[NASA]



[Hultsch, Fröhlich, 2023]

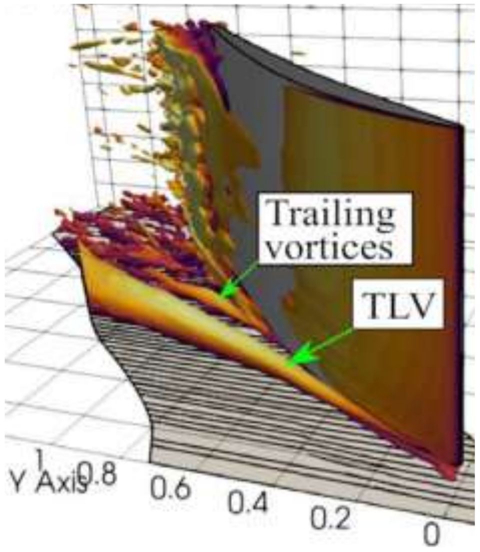


[hpctoday.com]



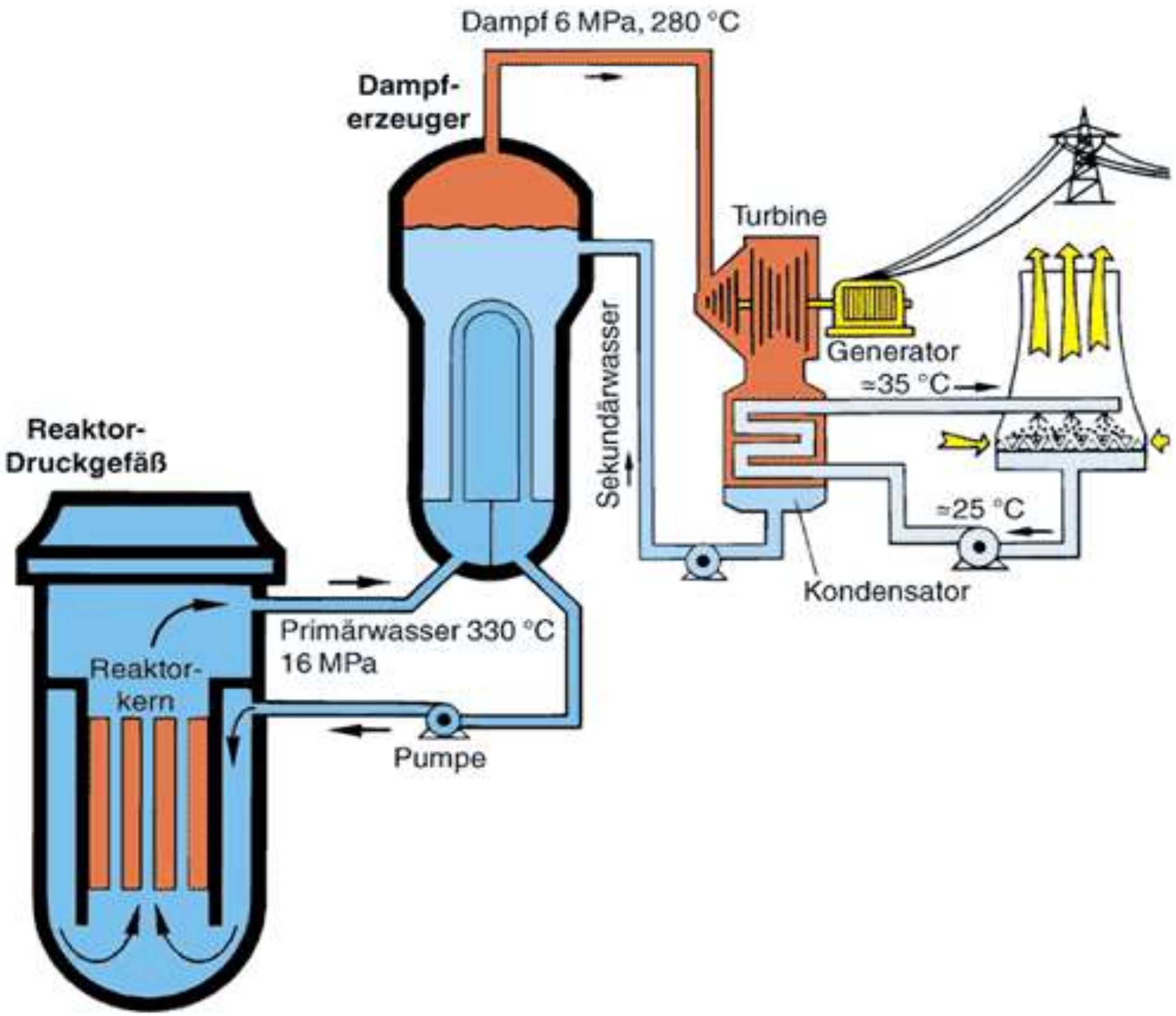
Power plants, chemical industry, …

Power plant: Turbulence in all parts



[Ventosa-Molina et al. 2021]

Example: flow around compressor blade



→ most technical flows are turbulent

Content

Description of turbulence

Modelling approaches for turbulent flow

RANS equations and closure problem

RANS models

RANS modelling near walls

LES and hybrids

Vortex detection

Final recommendations

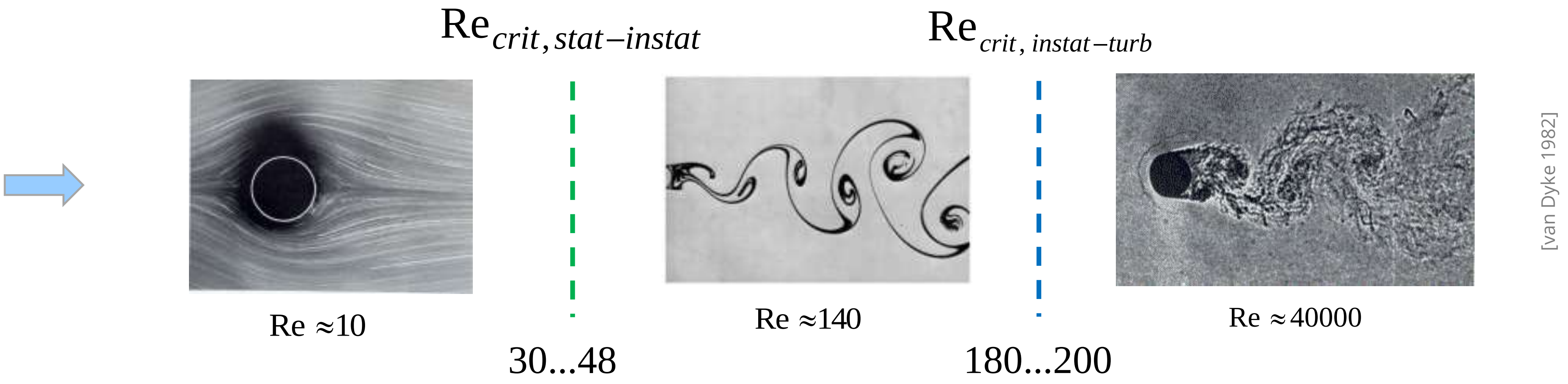
Here, in given time, only limited coverage

Generation of turbulence

Increasing Reynolds number, e.g. by increasing velocity

$$Re = \frac{U\,L}{\nu} = \frac{inertial\ forces}{viscous\ forces}$$

Instability (often sequence of instabilities) → Transition to turbulence



Properties of turbulent flows

- always unsteady and 3D
  - Laminar flows can be 1D, 2D, 3D
  - Laminar flows can be steady or unsteady

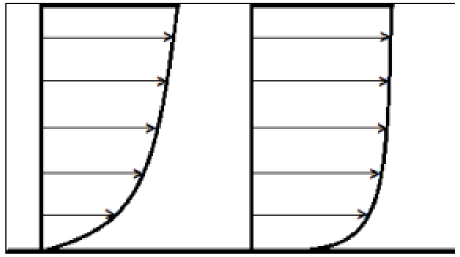
- irregular, „chaotic“
  - Laminar flows are regular and smooth

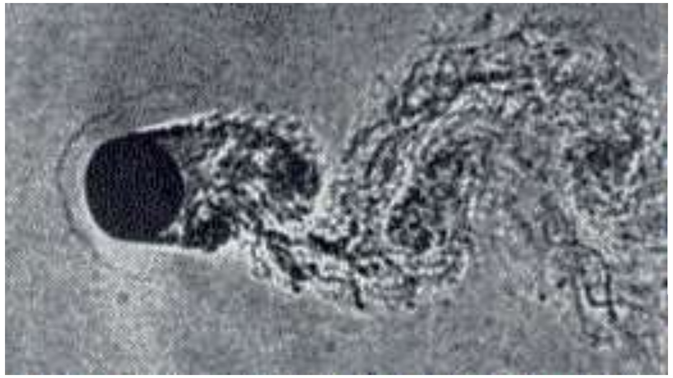
- enhanced exchange of momentum, heat, concentration, …

- more dissipation, more friction

- multiscale phenomenon

[van Dyke 1982]

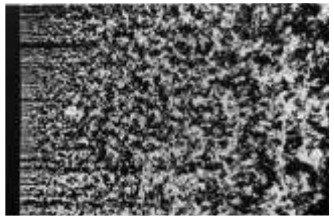




[van Dyke 1982]

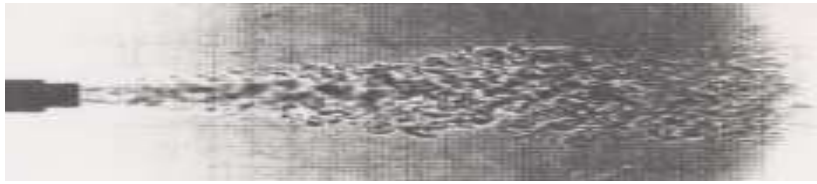
Different types of turbulent flow

- homogeneous & isotropic (statistically)



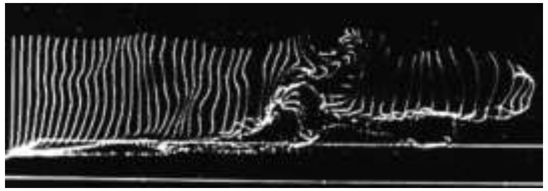
[van Dyke 1982]

- jets and shear layers

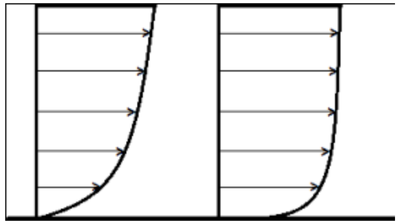


[van Dyke 1982]

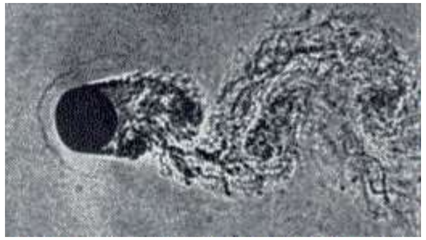
- boundary layers



[Kim et al. 1987]



- wakes behind bluff bodies

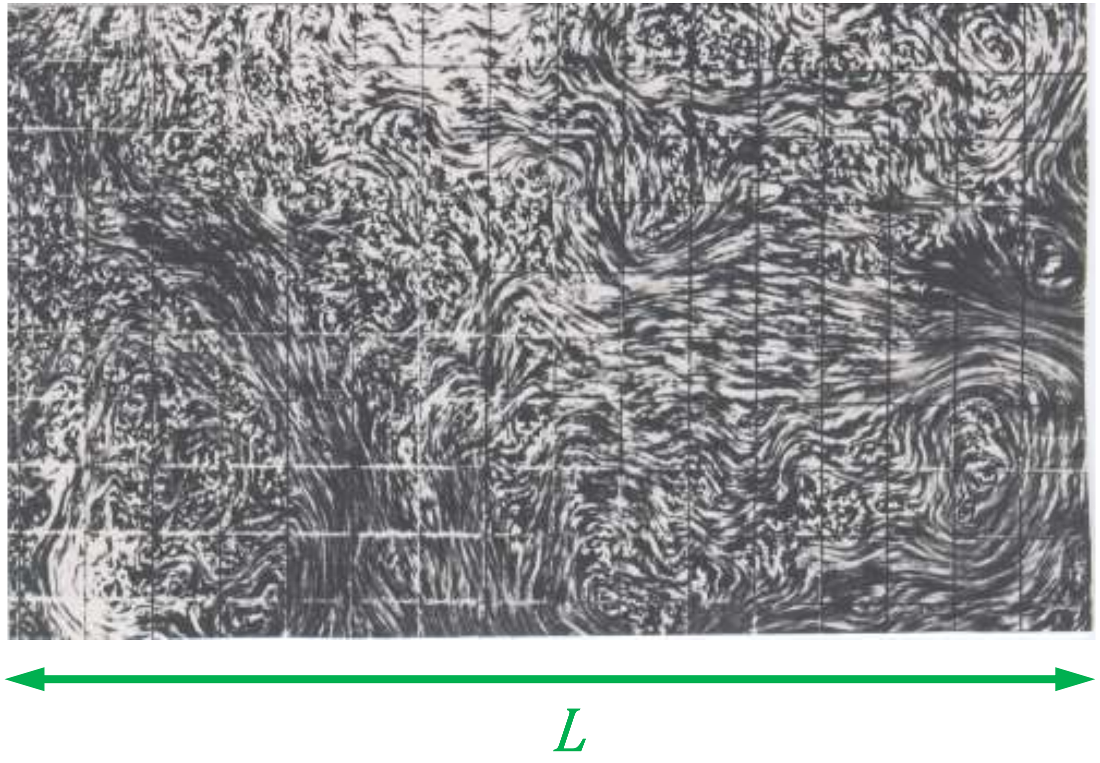


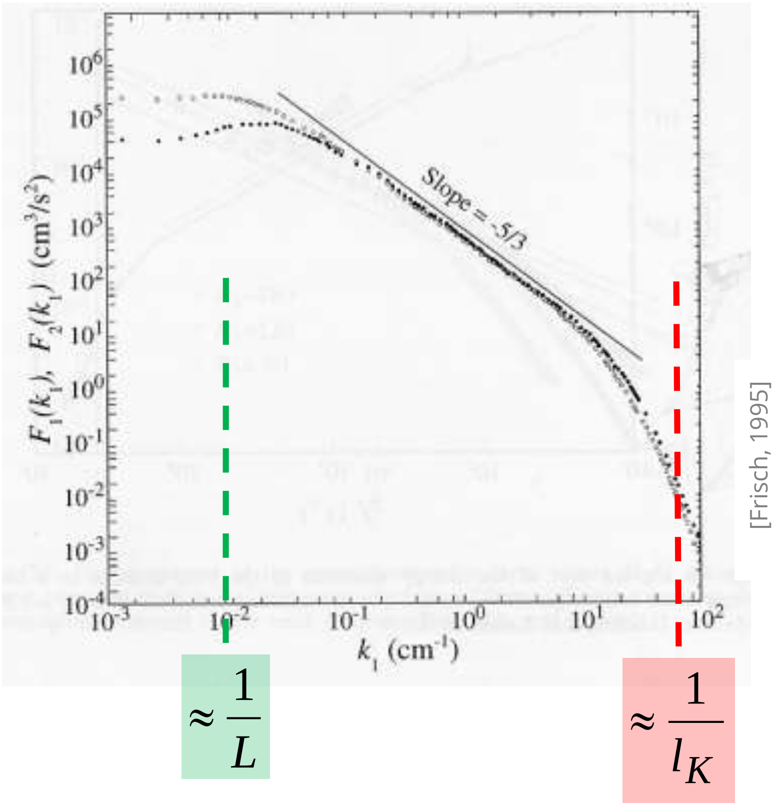
[van Dyke 1982]

- ....

real world turbulence is anisotropic

Turbulence is multiscale





- Continuous size distribution of fluctuating velocity contributions (spectrum)
  k = wave number

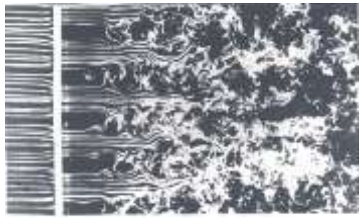
- NB: Holds for spectra in space and time (Taylor hypothesis)

Energy cascade [Kolmogorov 1941]

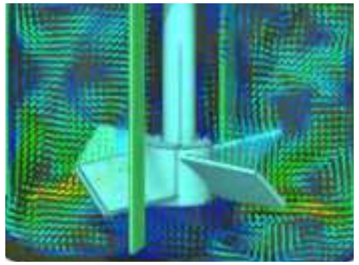
- Homogenous, isotropic turbulence
  Energy of eddies at wave number k

$$E(k) = P(k) - D(k)$$

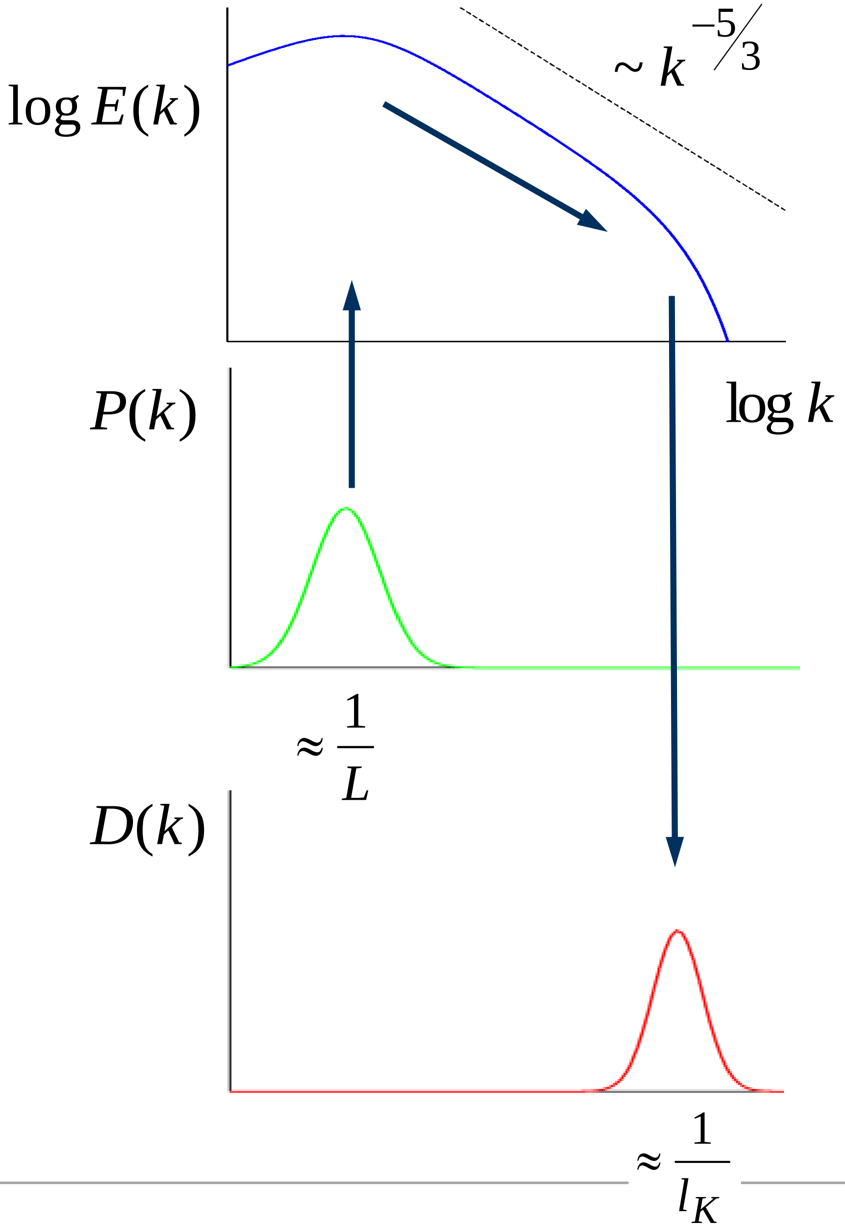
- Production P: large vortices
- Dissipation D: small vortices
- Inertial range: energy cascade



[van Dyke 1982]



[de Souza, 2013]



[Fröhlich, 2006]

- Theory [Pope 2000]

$$\frac{L}{l_K} \sim Re^{3/4}$$

- NB: Theory for statistical mean, not locally and instantanously

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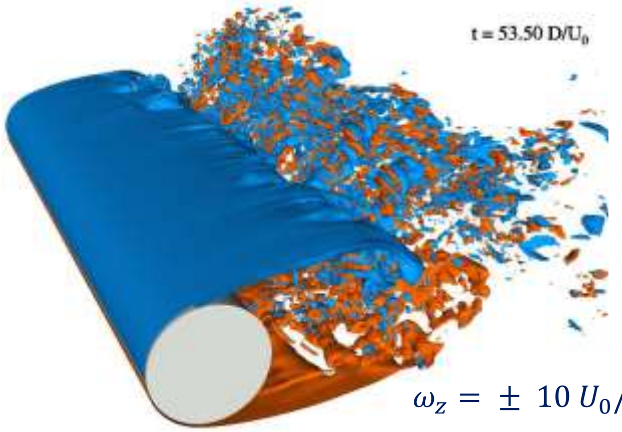
Navier-Stokes equations



Here, incompressible flow with $\rho = const.,\ \mu = const.,\ (\nu = \frac{\mu}{\rho} = const.)$

Conservation of mass $$\frac{\partial u_i}{\partial x_i} = 0$$

Conservation of momentum $$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial(\rho u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \underbrace{\frac{\partial \tau_{ij}}{\partial x_j}}_{= \mu \frac{\partial^2 u_i}{\partial x_j \partial x_j}}$$

$$\tau_{ij} = \mu\, S_{ij}$$

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

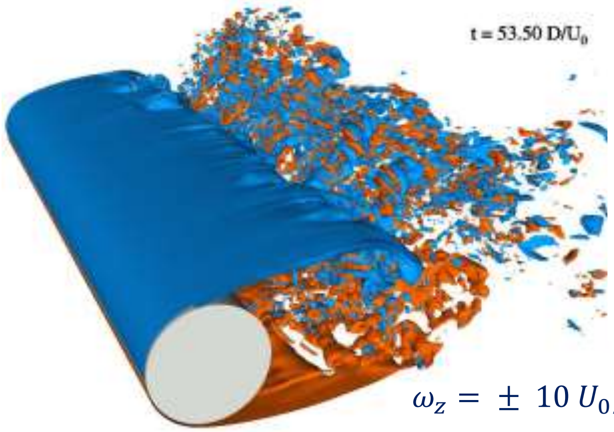
Equation of state $$\rho = \rho_0$$

Transport equation of scalar quantity c

$$\frac{\partial(\rho c)}{\partial t} + \frac{\partial(u_j \rho c)}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\Gamma_c \frac{\partial c}{\partial x_j}\right) + S_c$$

Navier-Stokes equations

Here, incompressible flow with $\rho = const.,\ \mu = const.,\ (\nu = \frac{\mu}{\rho} = const.)$



[Wissink, Rodi, 2008]

$\omega_z = \pm\ 10\, U_0/D$

Conservation of mass $$\frac{\partial u_i}{\partial x_i} = 0$$

Conservation of momentum $$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial(\rho u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j}$$

$$\tau_{ij} = \mu\, S_{ij}$$

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Fluid flow obeys NSE

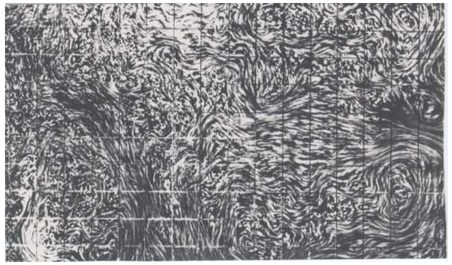
„Are the equations for turbulent flows the same as for laminar flows?“ → yes !

→ Why not just compute the unsteady turbulent flow?

Direct Numerical Simulation

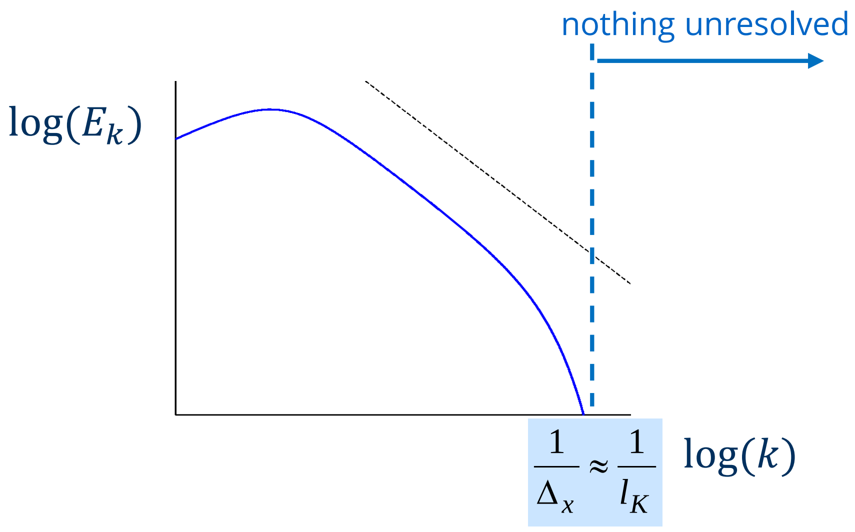
DNS

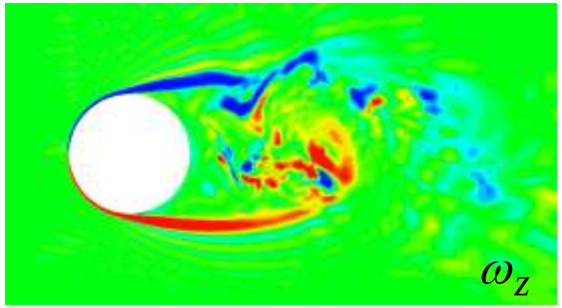
variable $u(x,t)$



equations

$$\frac{\partial u}{\partial t} + \frac{\partial uu}{\partial x} + \ \dots$$





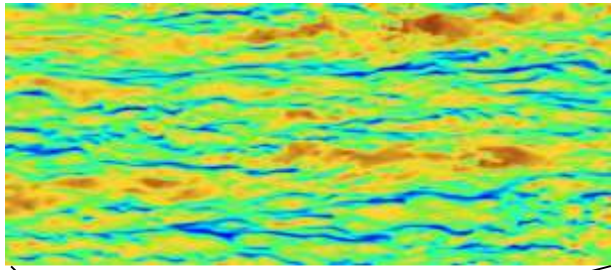
Turbulence fully resolved

very fine grid $N \sim Re^{9/4}$

very fine time step $N_t \sim Re^{3/4}$

cost: $t_{CPU} \sim Re^3$

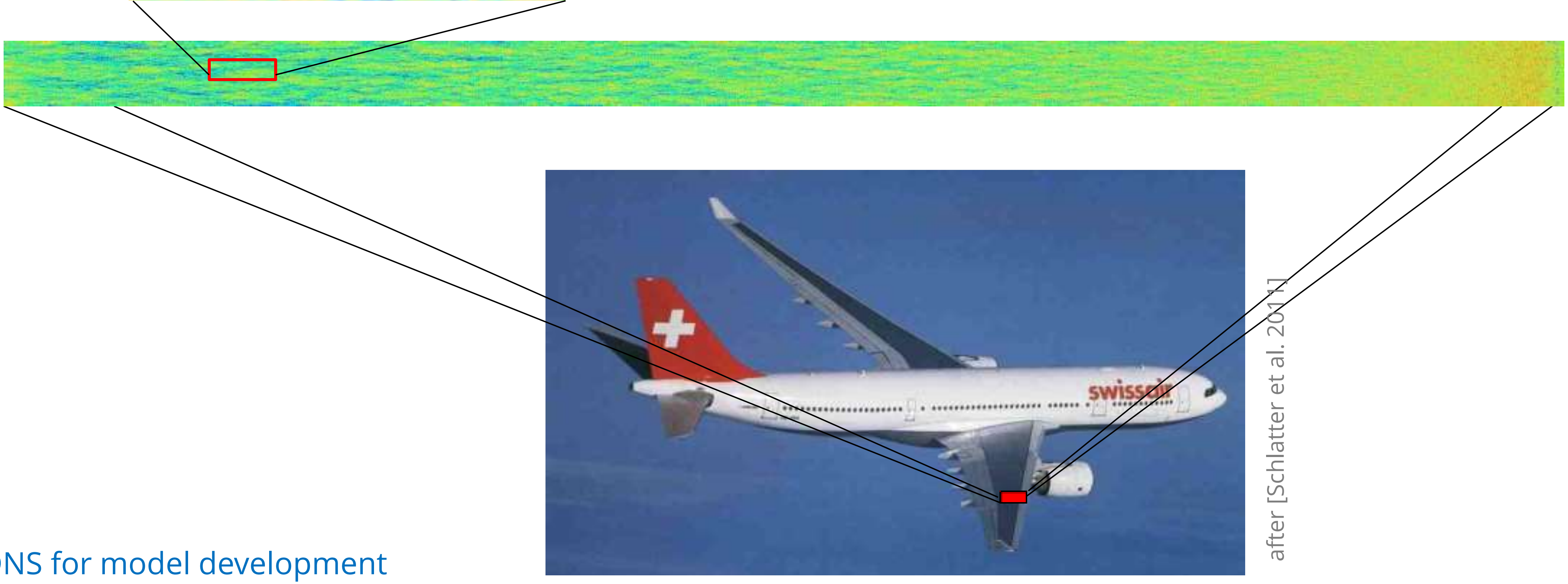
Direct Numerical Simulation



Even if DNS would be feasible

→ too costly for applications

→ overkill for typical questions



after [Schlatter et al. 2011]

→ DNS for model development

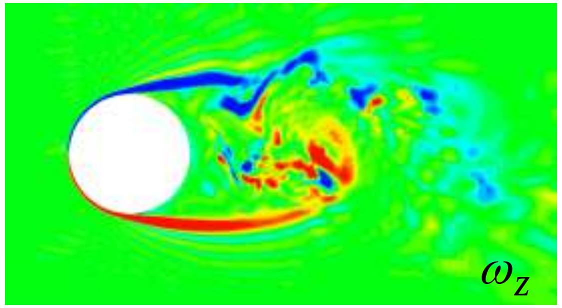
Reynolds-Averaged Navier Stokes (RANS) simulation

DNS

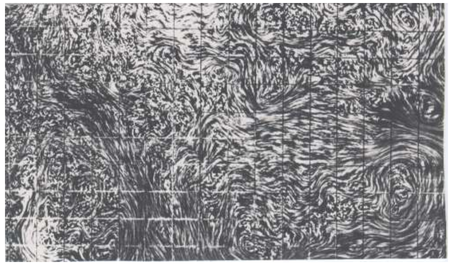
variable $u(x,t)$

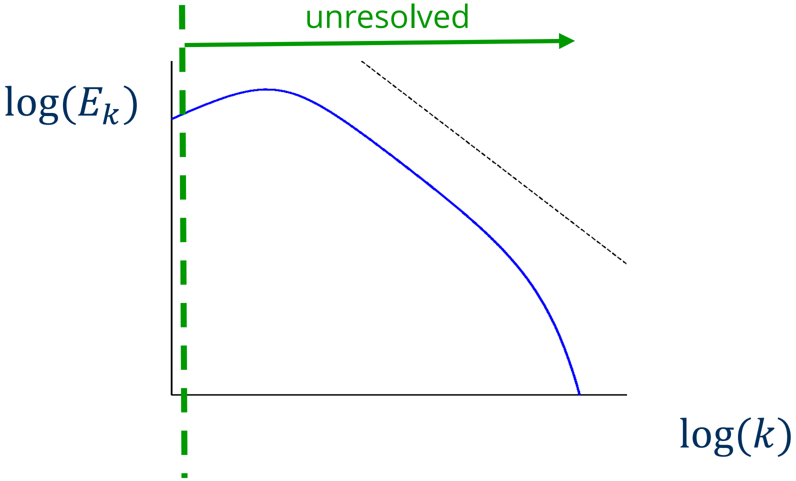
equations

$$\frac{\partial u}{\partial t} + \frac{\partial uu}{\partial x} + \ \dots$$



very fine grid $N \sim Re^{9/4}$





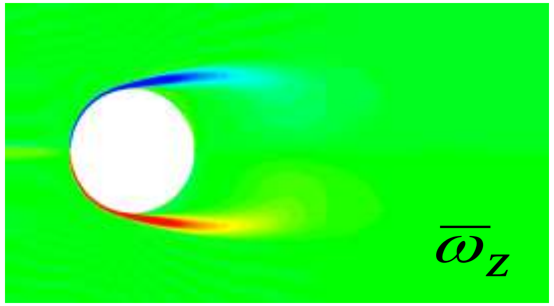
NO fluctuations resolved
only mean flow

RANS

$$\bar{u}(x) = \lim_{T\to\infty} \frac{1}{T} \int_0^T u(x,t)\,dt$$

$$u = \bar{u} + u'$$

$$\underbrace{\frac{\partial \bar{u}}{\partial t}}_{=0} + \frac{\partial \bar{u}\bar{u}}{\partial x} + \underbrace{\frac{\partial (\overline{u'u'})}{\partial x}}_{\text{unclosed}} + \ \dots$$



only coarse grid needed

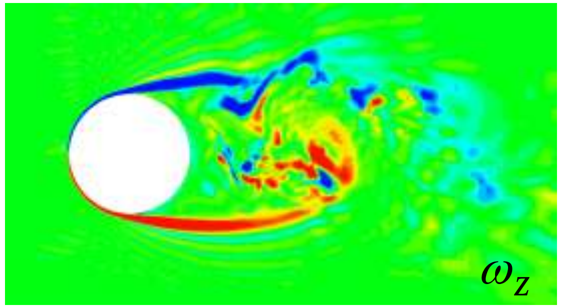
In between: Large Eddy Simulation (LES) and hybrid LES/RANS

DNS

variable $u(x,t)$

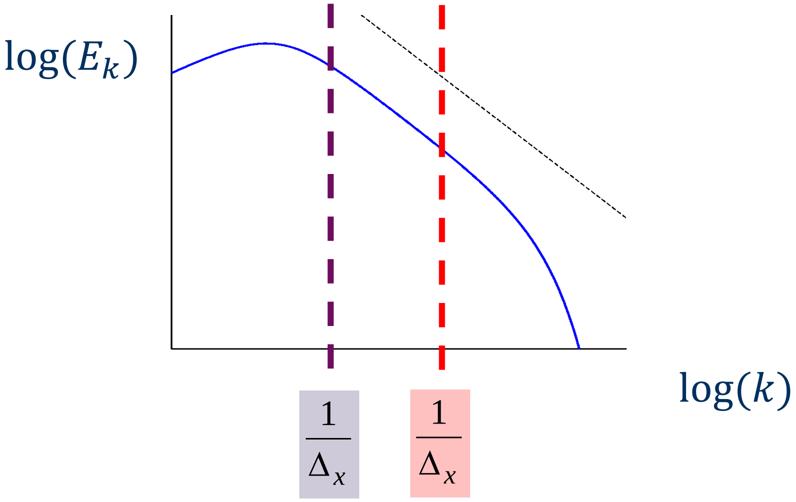
equations

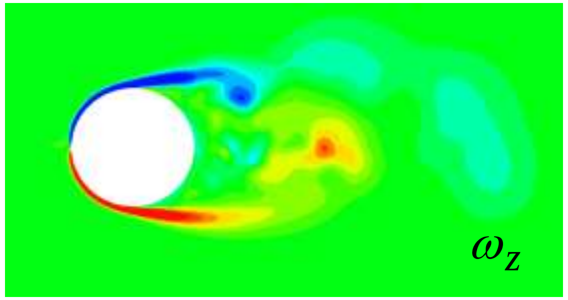
$$\frac{\partial u}{\partial t} + \frac{\partial uu}{\partial x} + \ \dots$$



very fine grid $N \sim Re^{9/4}$

LES and hybrids





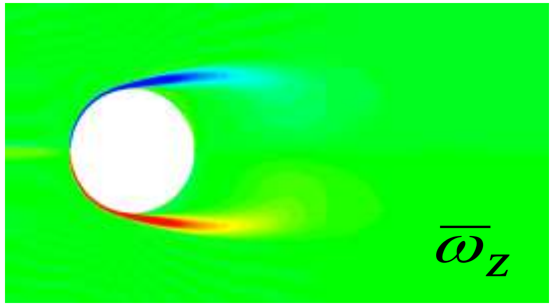
fine grid, unsteady simulation

RANS

$$\bar{u}(x) = \lim_{T\to\infty} \frac{1}{T} \int_0^T u(x,t)dt$$

$$u = \bar{u} + u'$$

$$\underbrace{\frac{\partial \bar{u}}{\partial t}}_{=0} + \underbrace{\frac{\partial \bar{u}\bar{u}}{\partial x} + \frac{\partial (\overline{u'u'})}{\partial x} + \ \dots}_{\text{unclosed}}$$



only coarse grid needed

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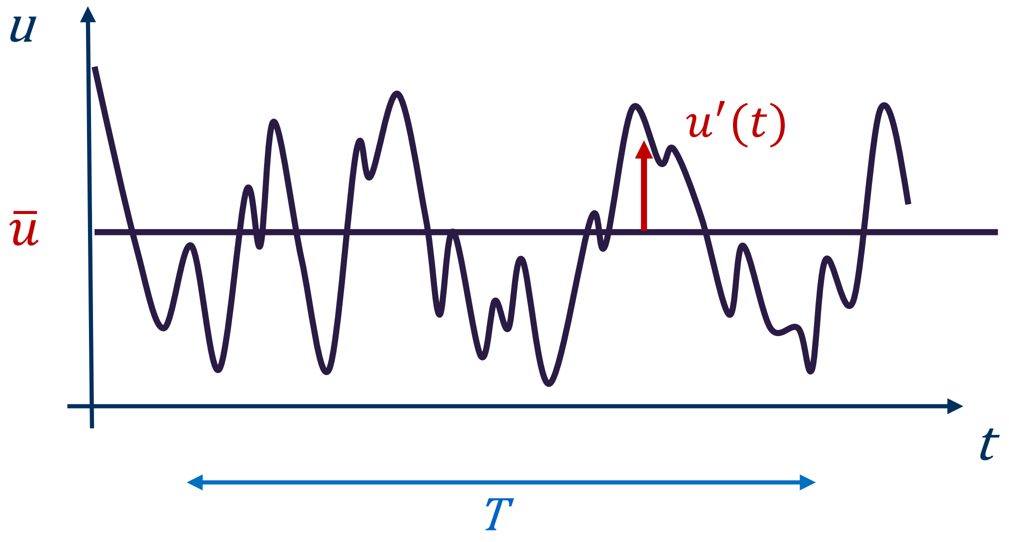
RANS - equations for mean flow

Reynolds averaging

Define „mean“ via averaging operation, e.g.

$$\bar{u}_i(x) = \lim_{T\to\infty} \frac{1}{T} \int_0^T u_i(x,t)dt$$

$$u_i = \bar{u}_i + u_i' \qquad p = \bar{p} + p' \qquad \text{etc.}$$



show that

$$\overline{a+b} = \bar{a} + \bar{b}\,, \quad \overline{\partial u_i/\partial x_j} = \partial\bar{u}_i/\partial\bar{x}_j$$

$$\overline{u_i'} = 0\,, \quad \bar{\bar{u}}_i = \bar{u}_i\,, \quad \overline{\bar{u}_i\, u_j'} = 0\,, \quad \overline{u_i' u_j'} \neq 0 \quad \text{(in general)}$$

compressible flows with

Favre averaging $\tilde{u}_i = \dfrac{\overline{\rho\, u_i}}{\overline{u_i}}$

Apply averaging to NSE

$$\overline{\frac{\partial \rho u_i}{\partial x_i}} = 0$$

$$\overline{\frac{\partial \rho u_i}{\partial t}} + \overline{\frac{\partial(\rho u_i u_j)}{\partial x_j}} = -\overline{\frac{\partial p}{\partial x_i}} + \overline{\frac{\partial \tau_{ij}}{\partial x_j}}$$

Equations for mean flow

Almost same equations as before

$$\frac{\partial \overline{u_i}}{\partial x_i} = 0$$

$$\frac{\partial \rho \overline{u_i}}{\partial t} + \frac{\partial (\rho\, \overline{u_i u_j})}{\partial x_j} = -\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j}$$

since $\rho = const.$:

$$\overline{\rho\, u_i} = \rho\, \overline{u_i}$$

$$\overline{\rho\, u_i u_j} = \rho\, \overline{u_i u_j}$$

$$\bar{u}_i(x) = \lim_{T\to\infty} \frac{1}{T}\int_0^T u_i(x,t)dt \neq f(t)$$

Convection term is non-linear

$$\overline{u_i\, u_j} = \overline{(\overline{u_i} + u_i')(\overline{u_j} + u_j')} = \overline{\overline{u_i}}\ \overline{\overline{u_j}} + \overline{u_i'\, \overline{u_j}} + \overline{\overline{u_i} u_j'} + \overline{u_i' u_j'} = \overline{u_i}\ \overline{u_j} + \overline{u_i' u_j'}$$

giving

$$\frac{\partial \overline{u_i}}{\partial x_i} = 0$$

new term

$$\frac{\partial (\rho \overline{u_i}\ \overline{u_j})}{\partial x_j} = -\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial\, \overline{\tau_{ij}}}{\partial x_j} - \frac{\partial \left(\rho\, \overline{u_i' u_j'}\right)}{\partial x_j}$$

Equations for mean flow

Re-writing

$$\frac{\partial \overline{u_i}}{\partial x_i} = 0$$

$$\frac{\partial(\rho\,\overline{u_i}\,\overline{u_j})}{\partial x_j} = -\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial\,\overline{\tau_{ij}}}{\partial x_j} - \frac{\partial\left(\rho\,\overline{u_i' u_j'}\right)}{\partial x_j} = -\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\overline{\tau_{ij}} + \tau_{ij}^R\right)$$

with

tensor of viscous stresses $\overline{\tau_{ij}} = 2\mu\overline{S_{ij}} = \mu\left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i}\right)$

Reynolds stress tensor $\tau_{ij}^R = -\rho\,\overline{u_i' u_j'}$

„Are the equations for turbulent flows the same as for laminar flows?“ → NO, there is another term !

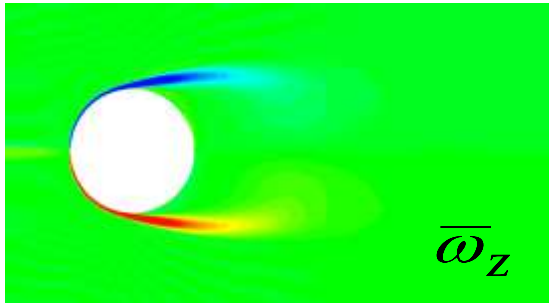
Now different description: want to know $\overline{u_i}$ instead of u_i

Closure problem

RANS equations

$$\frac{\partial \overline{u_i}}{\partial x_i} = 0$$

$$\frac{\partial(\rho \overline{u_i}\,\overline{u_j})}{\partial x_j} = -\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial\,\overline{\tau_{ij}}}{\partial x_j} - \frac{\partial\left(\rho\,\overline{u_i' u_j'}\right)}{\partial x_j}$$



+ initial and boundary conditions

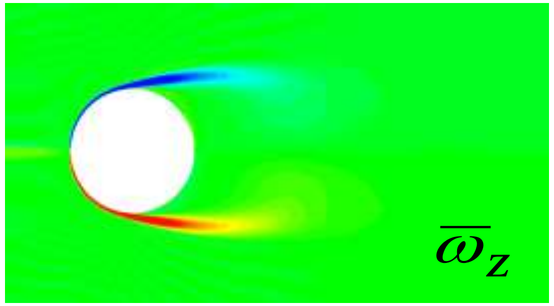
Problem describes mean flow (only)

Closure problem

RANS equations

$$\frac{\partial \overline{u_i}}{\partial x_i} = 0$$

$$\frac{\partial(\rho \overline{u_i}\,\overline{u_j})}{\partial x_j} = -\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial \overline{\tau_{ij}}}{\partial x_j} - \frac{\partial\left(\rho\,\overline{u_i' u_j'}\right)}{\partial x_j}$$



+ initial and boundary conditions

$$-\rho\,\overline{u_i' u_j'} = -\rho \begin{pmatrix} \overline{(u_1')^2} & \overline{u_1' u_2'} & \overline{u_1' u_3'} \\ \overline{u_2' u_1'} & \overline{(u_2')^2} & \overline{u_2' u_3'} \\ \overline{u_3' u_1'} & \overline{u_3' u_2'} & \overline{(u_3')^2} \end{pmatrix}$$

How to get $\overline{u_i' u_j'}$? (6 unknowns)

fluctuations u_i' are not present in this approach

term impossible to know without further info

equation system is not complete → cannot be solved as stated → equations are **„not closed"**

Further equations for closure?

$$-\rho\,\overline{u_i'u_j'} = -\rho\begin{pmatrix} \overline{(u_1')^2} & \overline{u_1'u_2'} & \overline{u_1'u_3'} \\ \overline{u_2'u_1'} & \overline{(u_2')^2} & \overline{u_2'u_3'} \\ \overline{u_3'u_1'} & \overline{u_3'u_2'} & \overline{(u_3')^2} \end{pmatrix}$$

Can devise equations for $\rho\overline{u_i'u_j'}$ $\quad i = 1,2,3 \quad j = 1,2,3$

$$\frac{\partial\left(\rho\overline{u_i'u_j'}\right)}{\partial t} + \frac{\partial\left(\rho\bar{u}_k\overline{u_i'u_j'}\right)}{\partial x_k}$$
$$=$$
$$\frac{\partial}{\partial x_k}\left(\mu\frac{\partial\overline{u_i'u_j'}}{\partial x_k}\right) - \frac{\partial}{\partial x_k}\left(\rho\overline{u_k'u_i'u_j'} + \overline{p'u_j'}\delta_{ik} + \overline{p'u_i'}\delta_{jk}\right) - \rho\overline{u_i'u_k'}\frac{\partial\bar{u}_j}{\partial x_k} - \rho\overline{u_j'u_k'}\frac{\partial\bar{u}_i}{\partial x_k} + \overline{p'\left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right)} - 2\mu\overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}}$$

red terms need to be modelled

Can devise equations for $\rho\overline{u_k'u_i'u_j'}$ etc.

$$\frac{\partial\left(\rho\overline{u_i'u_j'u_k'}\right)}{\partial t} + \frac{\partial\left(\rho\bar{u}_l\overline{u_i'u_j'u_k'}\right)}{\partial x_l} = \ldots\ldots \quad \rho\overline{u_i'u_j'u_k'u_l'} \quad \ldots\ldots$$

generates even higher correlations
→ **cannot get a closed set of equations**

Resort: devise a model for $\tau_{ij}^R = -\rho\,\overline{u_i'u_j'}$ → do „**turbulence modelling**", need to get 6 terms

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Eddy viscosity models (EVM)

RANS equations $$\frac{\partial \overline{u_i}}{\partial x_i} = 0$$

$$\frac{\partial(\rho \overline{u_i}\,\overline{u_j})}{\partial x_j} = -\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial \overline{\tau_{ij}}}{\partial x_j} - \frac{\partial\left(\rho\,\overline{u_i' u_j'}\right)}{\partial x_j} = -\frac{\partial \overline{P}}{\partial x_i} + \frac{\partial}{\partial x_j}\left((\mu + \mu_t)\left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i}\right)\right)$$

tensor of viscous stresses $$\overline{\tau_{ij}} = 2\,\mu\,\overline{S_{ij}} = \mu\left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i}\right)$$ μ fluid property

Idea (Josef Boussinesq): $$\tau_{ij}^R = -\rho\,\overline{u_i' u_j'} \overset{Mod.}{=} \mu_t\left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i}\right) - \rho\frac{2}{3}k\,\delta_{ij}$$ μ_t depends on flow

typically μ_t >> μ

6 unknowns → 1 unknown:

$$k = \frac{1}{2}\,\overline{u_i' u_i'}$$

Next step: $$\mu_t = ???$$

$$P = p + \rho\frac{2}{3}k$$

Eddy diffusivity

Transport of scalar c

$$\frac{\partial(\rho c)}{\partial t} + \frac{\partial(u_j \rho c)}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\Gamma_c \frac{\partial c}{\partial x_j}\right) + S_c$$

Exact equation for mean

$$\cancel{\frac{\partial(\rho \bar{c})}{\partial t}} + \frac{\partial(\rho \overline{u_i}\, \bar{c})}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\Gamma_c \frac{\partial c}{\partial x_j}\right) + \frac{\partial(\rho \overline{u_i' c'})}{\partial x_j} + S_c$$

Γ_c fluid property

Modelled equation for mean

$$\cancel{\frac{\partial(\rho \bar{c})}{\partial t}} + \frac{\partial(\rho \overline{u_i}\, \bar{c})}{\partial x_j} = \frac{\partial}{\partial x_j}\left((\Gamma_c + \Gamma_t) \frac{\partial c}{\partial x_j}\right) + \overline{S_c}$$

Γ_t depends on flow

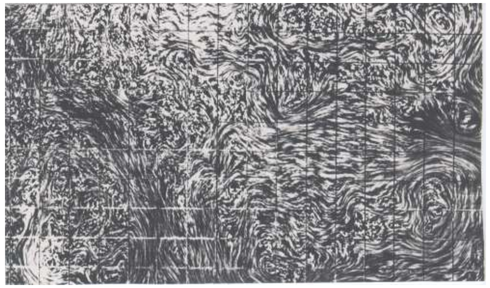
$$\Gamma_t = \frac{\mu_t}{\sigma_t}$$

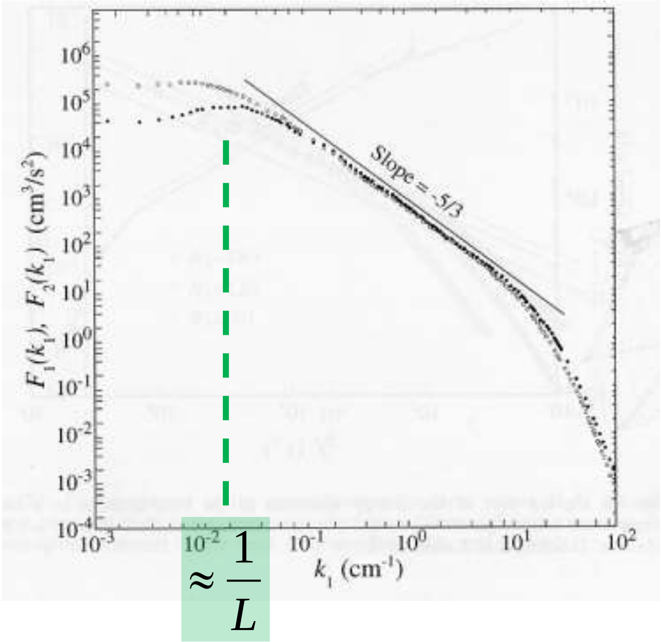
Eddy diffusivity:

turbulent diffusion Γ_t ← relate to → turbulent viscosity μ_t

exchange of mass via turb. fluct. — turb. Prandtl. no. σ_t — exchange of momentum via turb. fluct.

How to describe turbulence





- Size of eddies (turbulent length scale) $L_t \equiv L$
- Velocity of dominating eddies $U_t \equiv V$
- Turbulent kinetic energy (TKE) $K \equiv k$
- Dissipation rate ε
- Turbulent frequency $\omega = 1/T_t$
- ....

Theory of HIT: 2 quantities sufficient, but also minimum

Relations $V = K^{1/2}$, $\varepsilon = K^{3/2}/L$, $\omega = K^{1/2}/L$, . . .

Dimensional analysis

$$\mu_t = c\, VL$$

$$\mu_t = c_\mu\, \rho K^2 / \varepsilon$$

$$\mu_t = \rho K / \omega$$

......

→ Variety of models possible

Two-equation models

Example: $k-\varepsilon$ model

$$\mu_t = c_\mu \, \rho \frac{k^2}{\varepsilon}$$

$$P_k = -\rho \, \overline{u_i' u_j'} \frac{\partial \bar{u}_i}{\partial x_j}$$

Solve (modelled) equation for k

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial(\rho \bar{u}_j k)}{\partial x_j} = P_k - \rho\varepsilon + \frac{\partial}{\partial x_j}\left(\frac{\mu_t}{\sigma_k}\frac{\partial k}{\partial x_j}\right)$$

Solve (modelled) equation for ε

$$\frac{\partial(\rho \varepsilon)}{\partial t} + \frac{\partial(\rho \bar{u}_j \varepsilon)}{\partial x_j} = c_{\varepsilon 1}\frac{\varepsilon}{k} P_k - \rho c_{\varepsilon 2}\frac{\varepsilon^2}{k} + \frac{\partial}{\partial x_j}\left(\frac{\mu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_j}\right)$$

(with suitable BC)

5 empirical constants → take special cases, e.g. HIT, and get values of constants in these cases

→ two additional transport equation

→ then use μ_t in averaged momentum equation

Two-equation models

Example: $k-\omega$ model $\mu_t = \frac{\rho k}{\omega}$

$$P_k = -\rho\,\overline{u_i' u_j'}\,\frac{\partial \bar{u}_i}{\partial x_j}$$

Solve (modelled) equation for k

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial(\rho \bar{u}_j k)}{\partial x_j} = P_k - c_\mu \rho \omega + \frac{\partial}{\partial x_j}\left(\frac{\mu_t}{\sigma_k}\frac{\partial k}{\partial x_j}\right)$$

Solve (modelled) equation for ω

$$\frac{\partial(\rho \omega)}{\partial t} + \frac{\partial(\rho \bar{u}_j \omega)}{\partial x_j} = c_{\omega 1}\rho\frac{\omega}{k}P_k - \rho c_{\omega 2}\omega^2 + \frac{\partial}{\partial x_j}\left(\frac{\mu_t}{\sigma_\varepsilon}\frac{\partial \omega}{\partial x_j}\right)$$

(with suitable BC)

5 empirical constants → …

→ two additional transport equation

→ then use μ_t in averaged momentum equation

Two-equation models

Example: **Shear Stress Transport (SST) model** [Menter 1994]

$k - \varepsilon$ model

good away from walls

bad near walls

$k - \omega$ model

bad away from walls

good near walls

Combine both models with blending function

and reduce μ_t close to wall by **limiter**

→ Most widely used in practice today

Two-equation models

In practice

- Equations for $k, \varepsilon, \ldots$ more demanding than momentum equation (e.g. stability)
- Often different schemes than for momentum (more dissipative ones) to get convergence
- Often more iterations for these equations needed

Most widely used class of models in applications

Less than 2 transport equations for EVM ?

k-equation model of Prandtl $\mu_t = c\,V L$

1) $V = c_k \sqrt{k}$

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial(\rho \bar{u}_j k)}{\partial x_j} = P_k^{mod} - \varepsilon^{mod} + \frac{\partial}{\partial x_j}\left(\frac{\mu_t}{\sigma_k}\frac{\partial k}{\partial x_j}\right)$$

$$P_k = -\overline{u_i' u_j'}\,\overline{S_{ij}} \qquad P_k^{mod} = \mu_t 2\,\overline{S_{ij}}\,\overline{S_{ij}}$$

$$\varepsilon^{mod} = C_D\, k^{3/2} / L$$

2) Need to get L from empirical considerations

- $L = l_m$ called „mixing length“
- empirical, flow dependent
- outdated, but concept still used, e.g. near walls

Less than 2 transport equations for EVM ?

Spalart-Allmaras model

$$\mu_t = \rho\,\widetilde{\nu_t}\, f(y^+)$$

1 Transport equation for $\widetilde{\nu_t}$, modified eddy viscosity (better behavior near walls)

$$\frac{\partial(\widetilde{\nu_t})}{\partial t} + \frac{\partial(\rho \bar{u}_j \widetilde{\nu_t})}{\partial x_j} = \;\ldots.\; + \; f(d) \; + \ldots.$$

d = wall distance

No need for second equation

since ν_t computed by transport equation

Insert in momentum equation

Original version [Spalart, Allmaras 1994]

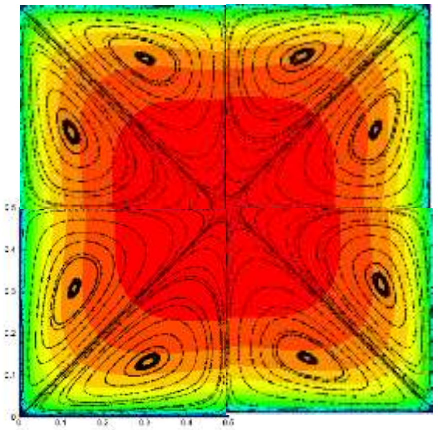
modifications & improvements in literature

EVM models

EVM inherently isotropic

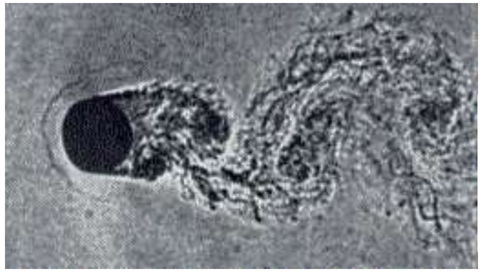
$$-\rho\,\overline{u_i'u_j'} = -\rho \begin{pmatrix} \overline{(u_1')^2} & \overline{u_1'u_2'} & \overline{u_1'u_3'} \\ \overline{u_2'u_1'} & \overline{(u_2')^2} & \overline{u_2'u_3'} \\ \overline{u_3'u_1'} & \overline{u_3'u_2'} & \overline{(u_3')^2} \end{pmatrix} = \underbrace{2\,\mu_t\,\overline{S_{ij}}} - \rho\frac{2}{3}k\,\delta_{ij}$$

μ_t same for all S_{ji}



Attention: some effects cannot be covered by EVM - by construction

- Example: secondary flows generated by turbulence (2nd kind)
- Steady RANS model not suited for large-scale unsteady flows (vortex shedding)
- Check that model is suitable for flow considered



[van Dyke 1982]

More than 2 equations for closure - RSM

$$-\rho\,\overline{u_i'u_j'} = -\rho\begin{pmatrix} \overline{(u_1')^2} & \overline{u_1'u_2'} & \overline{u_1'u_3'} \\ \overline{u_2'u_1'} & \overline{(u_2')^2} & \overline{u_2'u_3'} \\ \overline{u_3'u_1'} & \overline{u_3'u_2'} & \overline{(u_3')^2} \end{pmatrix}$$

Can devise equations for $\rho\overline{u_i'u_j'}$ $\quad i = 1,2,3 \quad j = 1,2,3$

$$\frac{\partial\left(\rho\overline{u_i'u_j'}\right)}{\partial t} + \frac{\partial\left(\rho\bar{u}_k\overline{u_i'u_j'}\right)}{\partial x_k} =$$

$$\frac{\partial}{\partial x_k}\left(\mu\frac{\partial\overline{u_i'u_j'}}{\partial x_k}\right) - \frac{\partial}{\partial x_k}\left(\rho\overline{u_k'u_i'u_j'} + \overline{p'u_j'}\delta_{ik} + \overline{p'u_i'}\delta_{jk}\right) - \rho\overline{u_i'u_k'}\frac{\partial\bar{u}_j}{\partial x_k} - \rho\overline{u_j'u_k'}\frac{\partial\bar{u}_i}{\partial x_k} + \overline{p'\left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right)} - 2\mu\overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}}$$

red terms need to be modelled

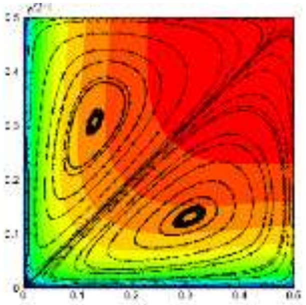
Provide closure models for all the red terms → „**Reynolds Stress Model**“ (**RSM**)

solve 6 equations for $\rho\overline{u_i'u_j'}$, closures need ε → solve additional PDE for ε

Properties

(–) More costly, less stable than EVM

(+) More general than EVM (e.g. swirl flows, secondary flows of second kind, …)



Hierarchy of RANS models

In addition to averaged eq. for mass and momentum

solve

0-equation model: algebraic relation for μ_t

1-equation model : one partial differential eq. → μ_t

2-equation model: two partial differential eq. → μ_t

$$-\rho\,\overline{u_i u_j} \overset{Mod}{=} \mu_t\,(\ldots\ldots) - \ldots$$

→ to momentum eq.

Reynolds Stress Model: 6 PDEs for $\overline{u_i' u_j'}$ and 1 for ε → $-\rho\,\overline{u_i' u_j'}$

For each class HUGE number of variants

Choice of model highly depends on flow, research question, desired accuracy, …

always do validation with similar case of reference

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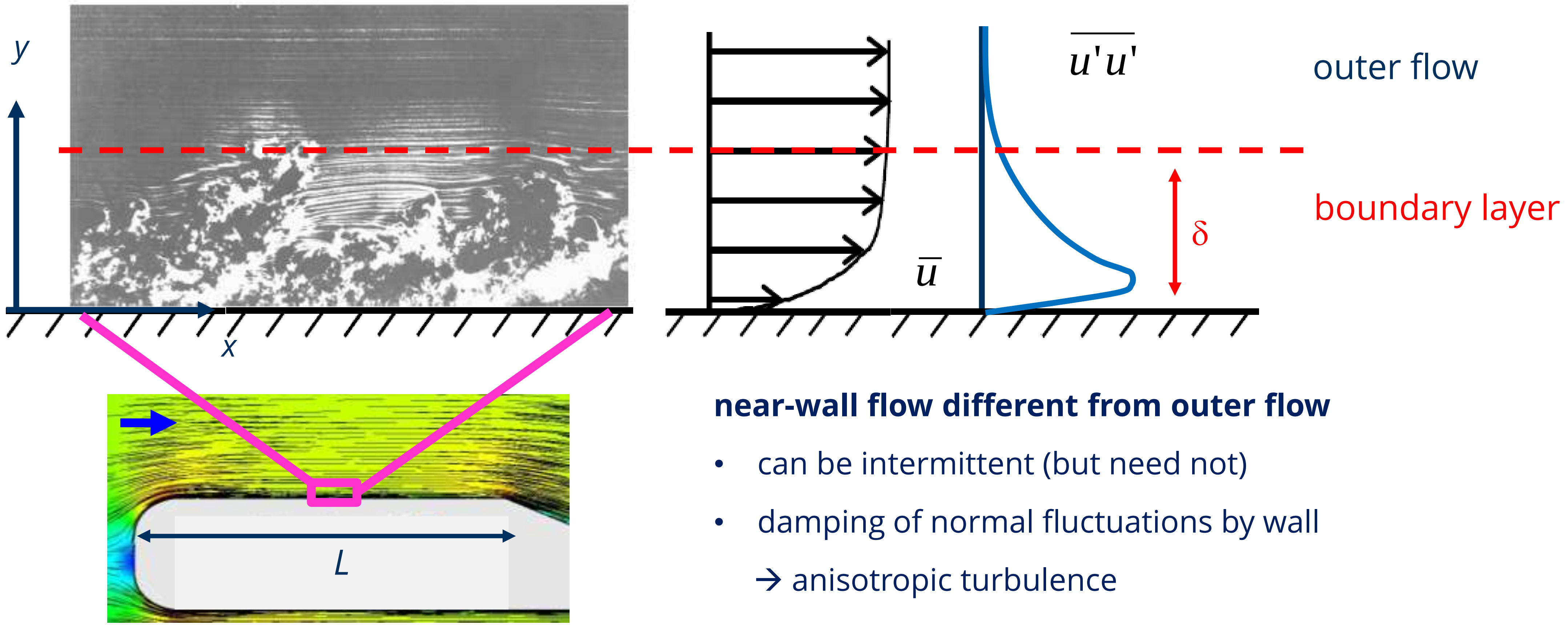
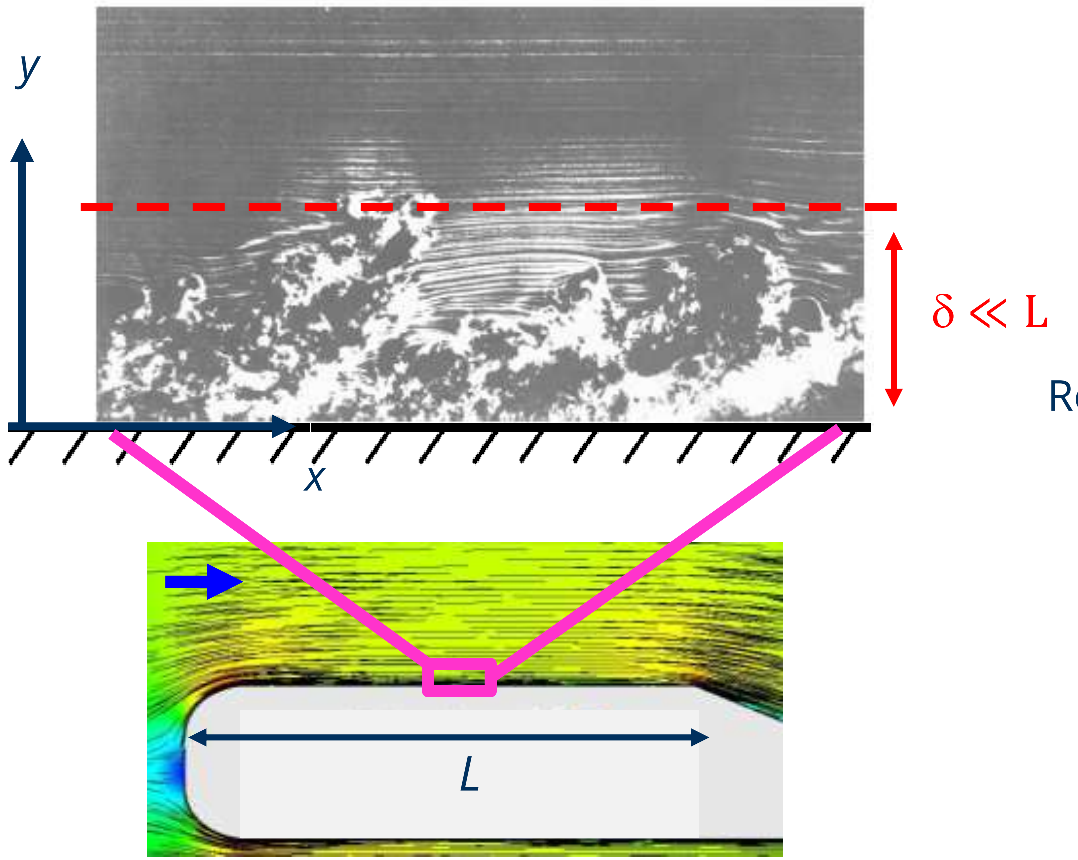
Near-wall turbulence



near-wall flow different from outer flow

- can be intermittent (but need not)
- damping of normal fluctuations by wall
  → anisotropic turbulence
- very close to wall viscous effects dominant
- very thin BL for high Re: $\delta / x = 0.371\, Re_x^{-1/5}$
- fine-scale turbulence for high Re

Near-wall turbulence

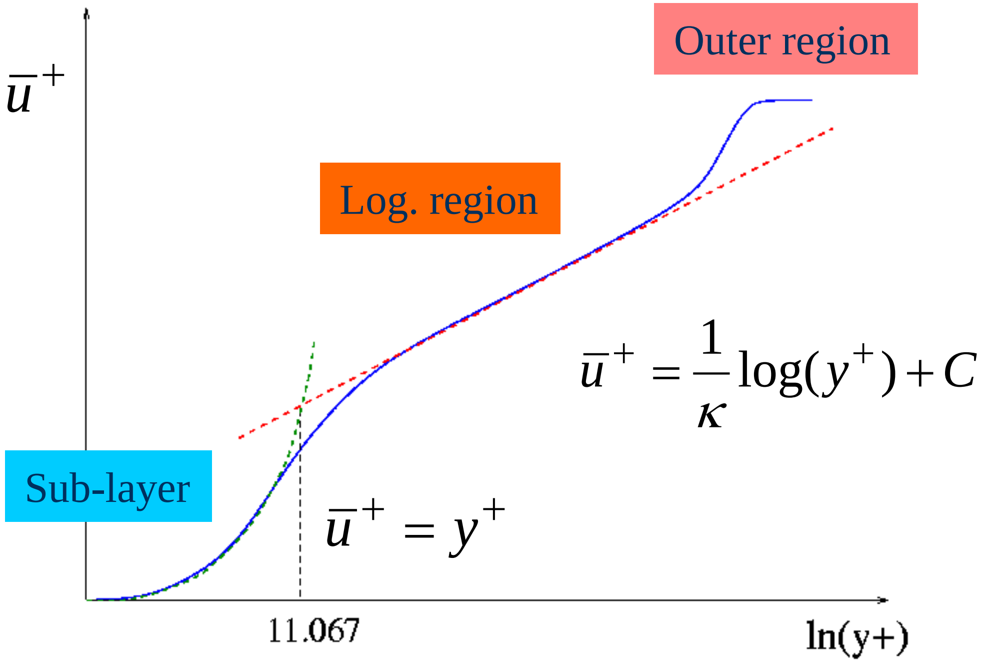


Turbulence near wall is different

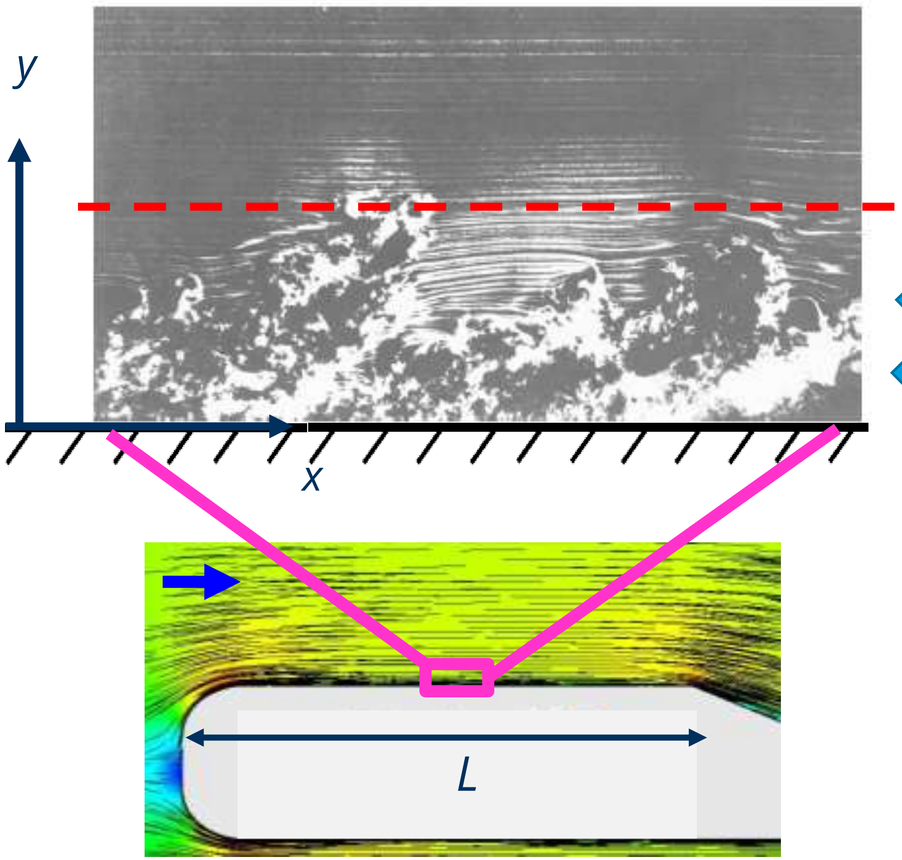
velocity scale $u_\tau = \sqrt{\frac{\tau_w}{\rho}} \rightarrow u^+ = \frac{u}{u_\tau}$

reference scale $l_\tau = \frac{\nu}{u_\tau} \rightarrow y^+ = \frac{y}{l_\tau}$

Reference case $dp/dx = 0$

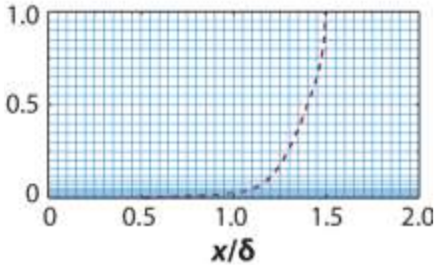


Near-wall turbulence



Turbulent fluctuations

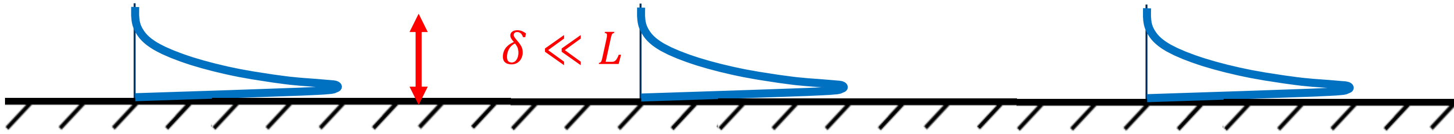
- small scale near wall
- very **fine** grid in **all** directions for DNS etc.



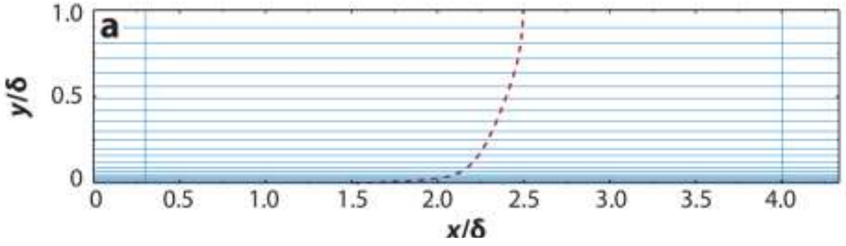
[Waldmann, Schollenberger, 2023]

Statistics

- change slowly along the wall $\sim L$
- change rapidly normal to wall $\sim \delta$



- **coarse** grid **parallel** to wall
  for RANS simulations
- Denser close to wall



Wall treatment – 1

Resolve statistics

Refine grid near wall   (generates large grids and higher cost)

First grid point at $y^+ \approx 1$

Turbulence is different (local Reynolds number $y^+ = yu_\tau/\nu$ is small)

→ **modify turbulence model**, e.g. $k - \varepsilon$ model

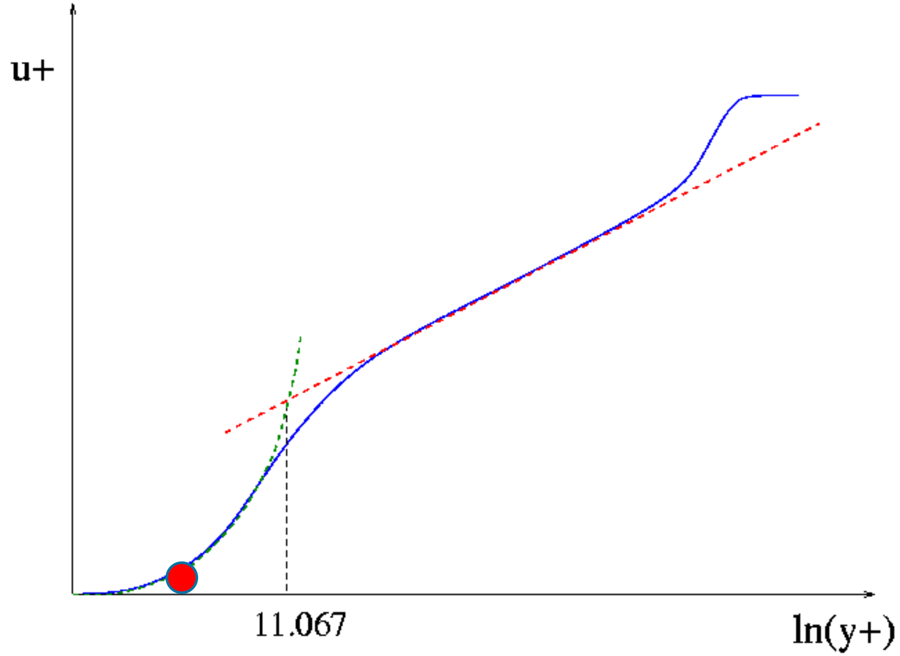
introduce damping functions for some terms

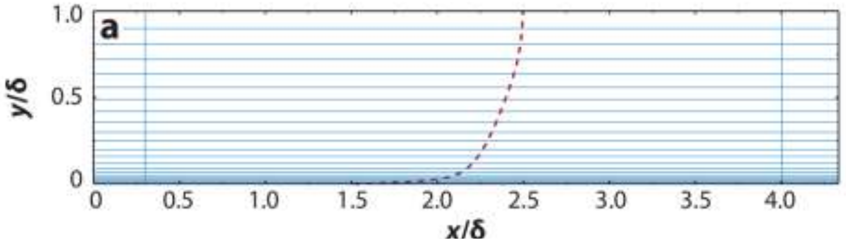
models called „**low Reynolds number models**"

some turbulence models can do without modification

e.g. $k - \omega$ model

just use fine grid





Wall treatment – 2

Model part of near-wall statistics

Do not resolve the near-wall part of the BL

First grid point at $y^+ > 30$ ….

local Reynolds number $y^+ = yu_\tau/\nu$ is large

→ use special conditions at 1st point

so-called **wall functions (WF)**

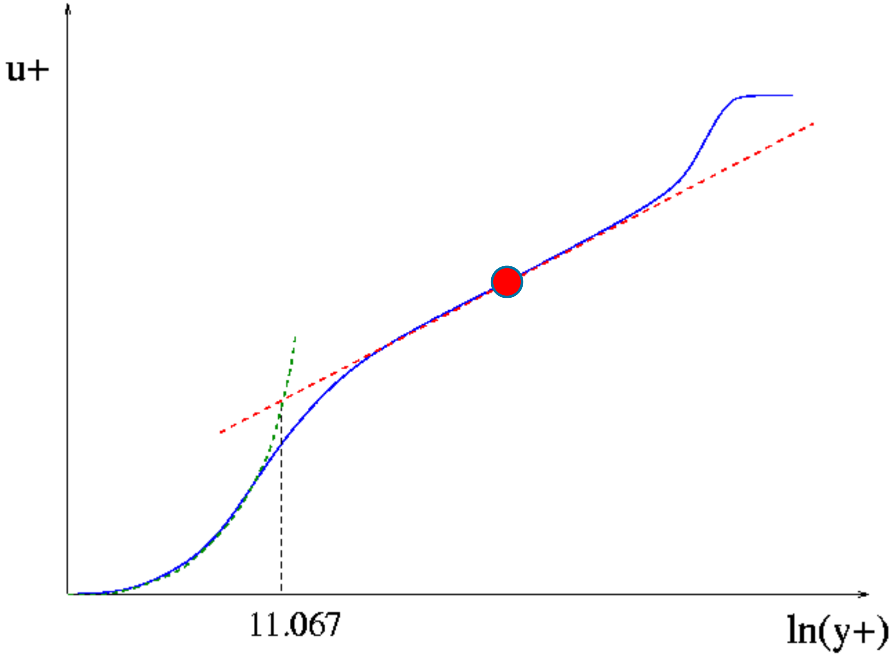
WF contain statistical information

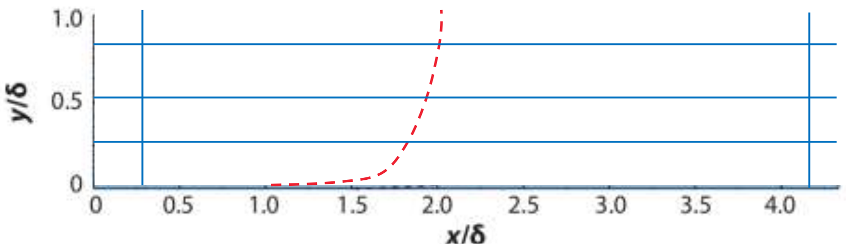
grid much coarser normal to wall

lower cost

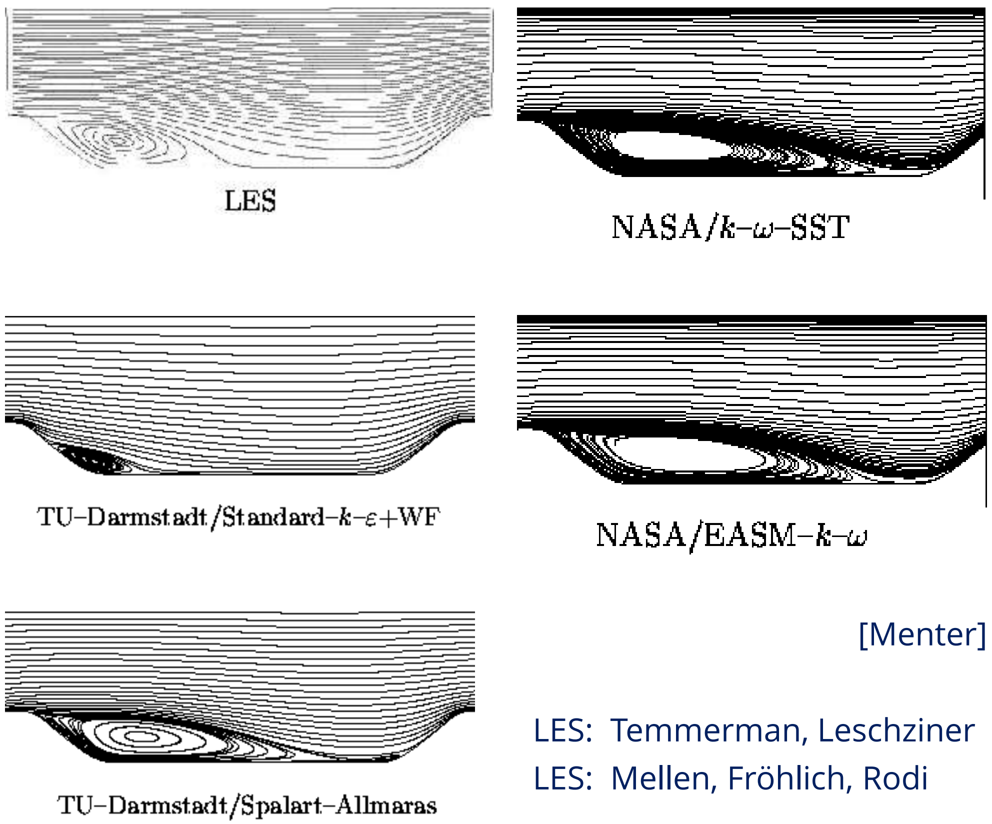
More modelling → potentially higher modelling error

For very high *Re* no other choice





Example: periodic hill flow [Mellen, Fröhlich, Rodi 2000]



[Menter]

LES: Temmerman, Leschziner

LES: Mellen, Fröhlich, Rodi

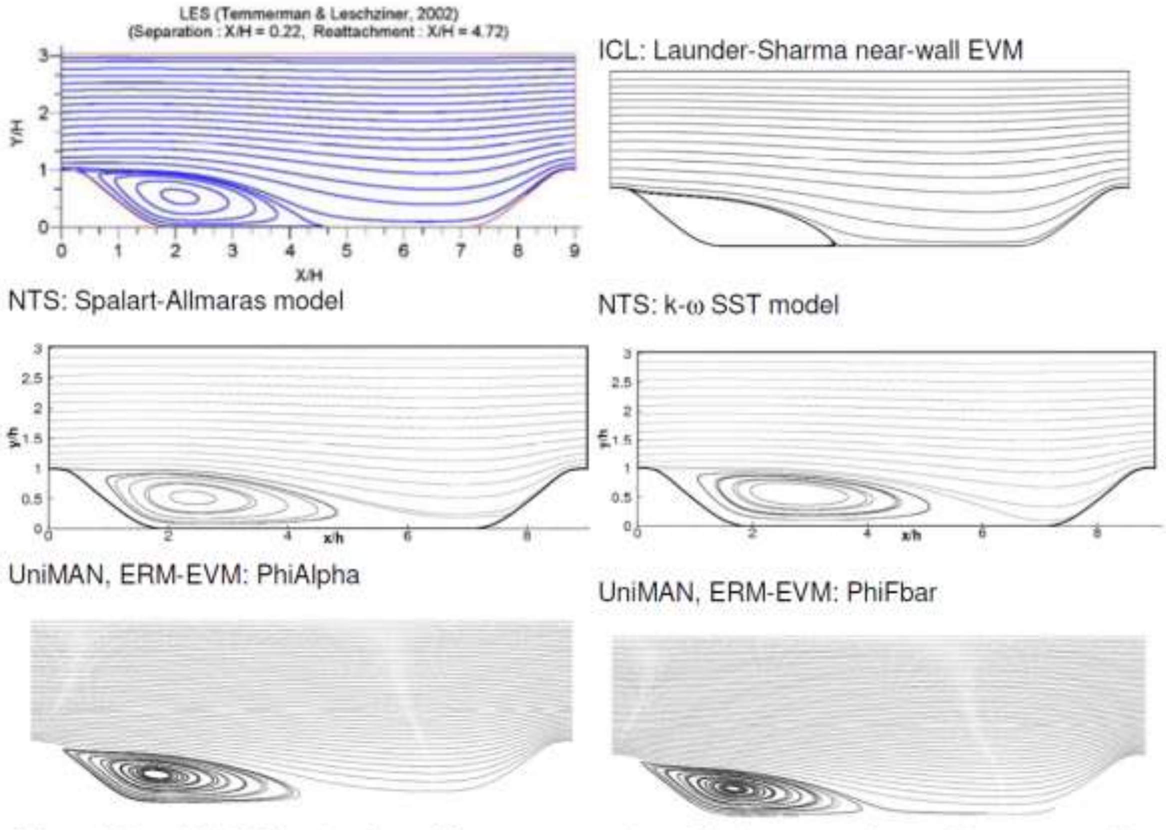


Figure 9: Re_H=10600, Eddy-viscosity model group – comparison of the time-averaged streamline patterns with the reference one (top left)

[Jakirlic, 2012]

→ for THIS case Problems with all RANS models, even RSM

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after [Fröhlich 2006]

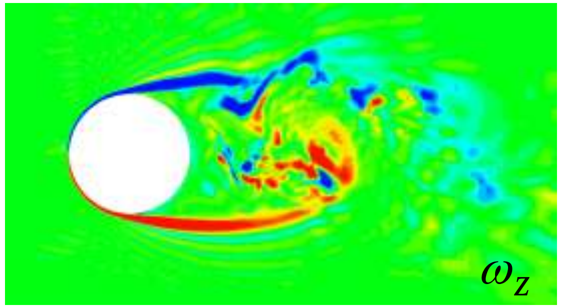
In between: Large Eddy Simulation (LES) and hybrid LES/RANS

DNS

variable $u(x,t)$

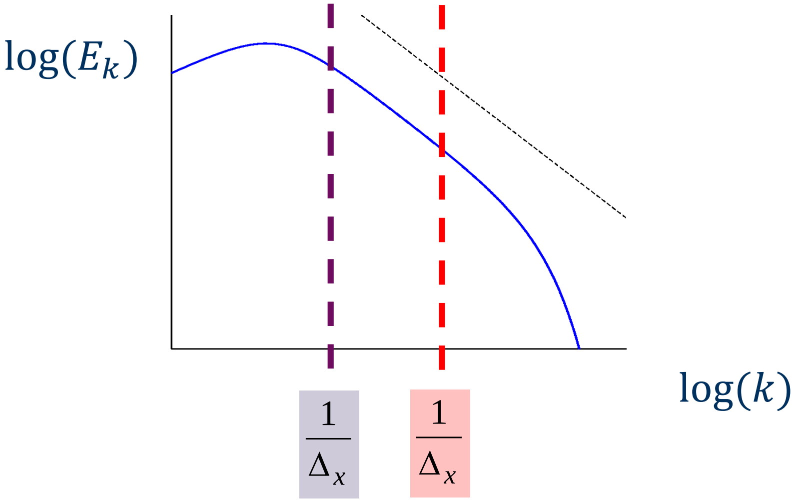
equations

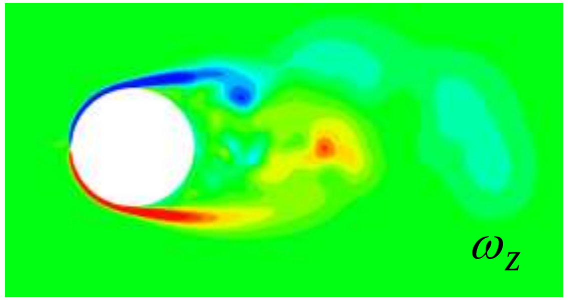
$$\frac{\partial u}{\partial t} + \frac{\partial uu}{\partial x} + \ \ldots$$



very fine grid $N \sim Re^{9/4}$

LES and hybrids





fine grid, unsteady simulation

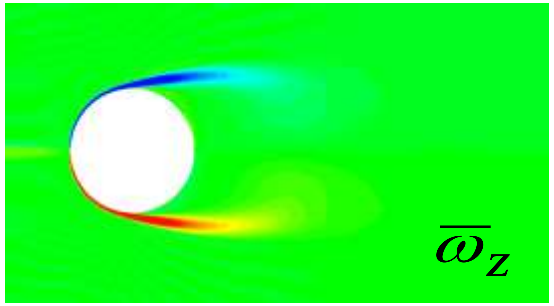
RANS

$$\bar{u}(x) = \lim_{T\to\infty} \frac{1}{T} \int_0^T u(x,t)dt$$

$$u = \bar{u} + u'$$

$$\underbrace{\frac{\partial \bar{u}}{\partial t}}_{=0} + \frac{\partial \bar{u}\bar{u}}{\partial x} + \underbrace{\frac{\partial (\overline{u'u'})}{\partial x}}_{\text{turbulence model}} + \ \ldots$$

=0 turbulence model



only coarse grid needed

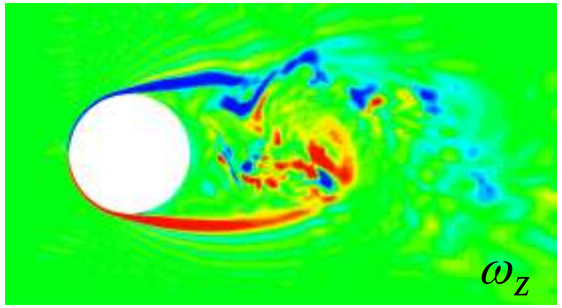
In between: Large Eddy Simulation (LES) and hybrid LES/RANS

DNS

variable $u(x,t)$

equations

$$\frac{\partial u}{\partial t} + \frac{\partial uu}{\partial x} + \ \dots$$



very fine grid $N \sim Re^{9/4}$

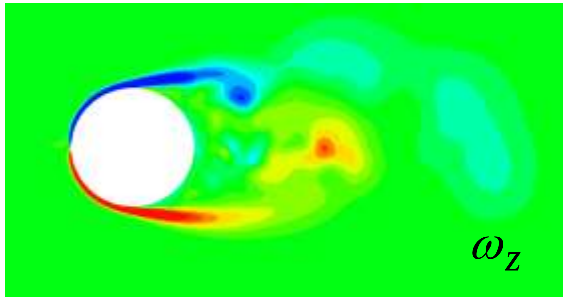
LES

$$\bar{u}(x,t) = \int_{\Omega} G_\Delta(\xi - x) u(\xi,t) d\xi$$

$u = \bar{u} + u'$ filter = smoother

$$\frac{\partial \bar{u}}{\partial t} + \frac{\partial \bar{u}\bar{u}}{\partial x} + \underbrace{\frac{\partial (\overline{uu} - \bar{u}\,\bar{u})}{\partial x}} \dots$$

subgrid-scale model



fine grid, unsteady simulation

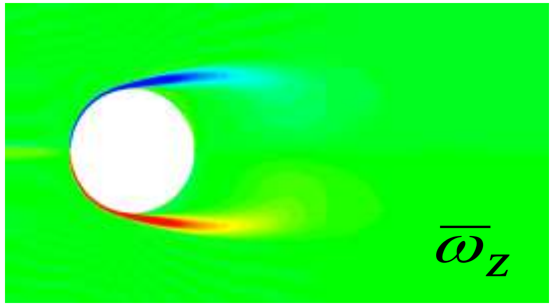
RANS

$$\bar{u}(x) = \lim_{T\to\infty} \frac{1}{T} \int_0^T u(x,t)dt$$

$u = \bar{u} + u'$ average

$$\underbrace{\frac{\partial \bar{u}}{\partial t}}_{=0} + \frac{\partial \bar{u}\bar{u}}{\partial x} + \underbrace{\frac{\partial (\overline{u'u'})}{\partial x}} + \ \dots$$

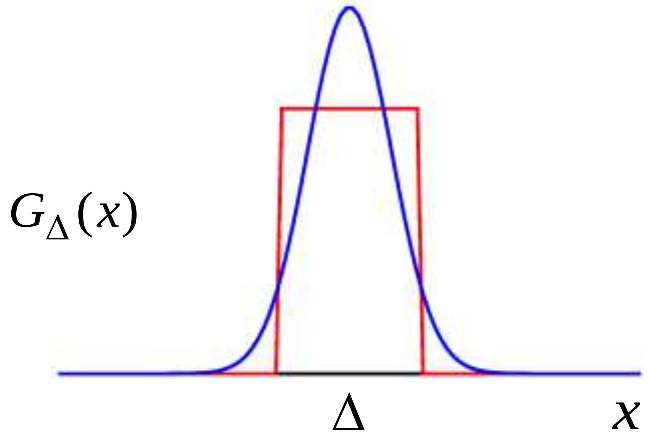
turbulence model



only coarse grid needed

LES equations

- Filter $$\bar{u}(x,t) = \int G_\Delta(x-\xi)\ u(\xi,t)\ d\xi$$
- Filter in space, not time
- $\bar{\bar{u}} \neq \bar{u} \quad \overline{u'} \neq 0 \quad \overline{\bar{u}v} \neq \bar{u}\bar{v}$
- For $\Delta \to 0 \quad \bar{u}(x) \to u(x)$
- If $\Delta = const$, commutes with derivative $$\overline{\frac{\partial u}{\partial x}} = \frac{\partial \bar{u}}{\partial x}$$
- Equation for large scales



$$\frac{\partial \bar{u}_i}{\partial x_i} = 0$$

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j}(\bar{u}_i\bar{u}_j) + \frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} = \nu \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j} - \frac{\partial}{\partial x_j}(\overline{u_i u_j} - \bar{u}_i\bar{u}_j)$$

term unclosed
→ subgrid-scale model
→ e.g. eddy viscosity

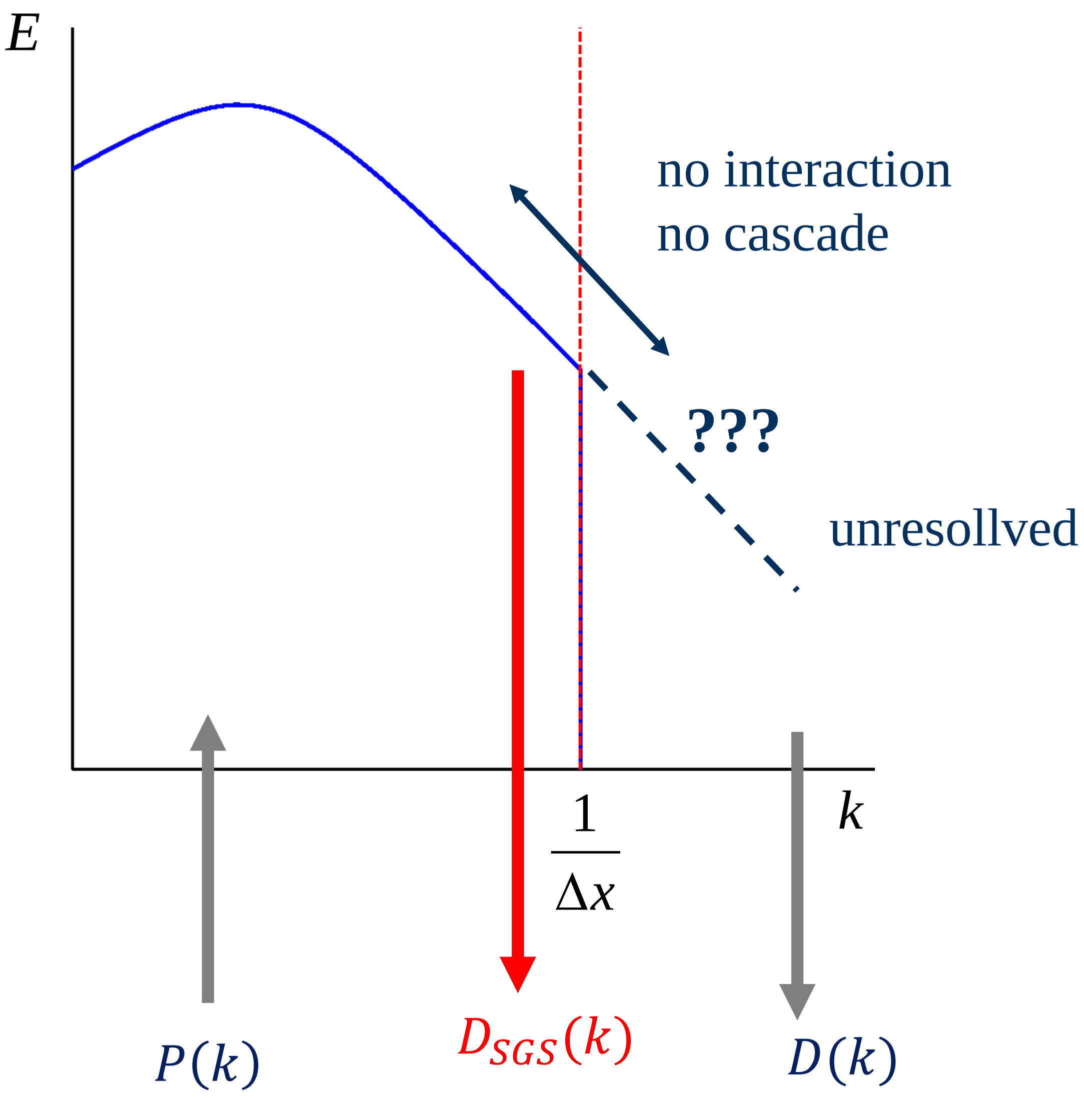
LES modelling

- Energy transfer

$$E(k) = P(k) - D(k)$$

- Large scales
  - Large, energy carrying vorctices
  - resolved by grid
  - Production P resolved

- Small scales (“sub-grid scale”, SGS)
  - Small eddies, little energy
  - dissipate energy
  - too small for grid, not resolved
  - main role of SGS model: dissipation at right amount



Subgrid-scale modelling & hybrid

Unclosed term $\tau_{ij} = \left(\overline{u_i u_j} - \bar{u}_i \bar{u}_j\right)$

Simplest model is eddy model of Smagorinski (1963)

$$\tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk} \quad \approx \quad \tau_{ij}^{SM} = -\nu_t \, 2\, \overline{S_{ij}}$$

algebraic relation

$$\nu_t = (C_S\Delta)^2 |\bar{S}|$$

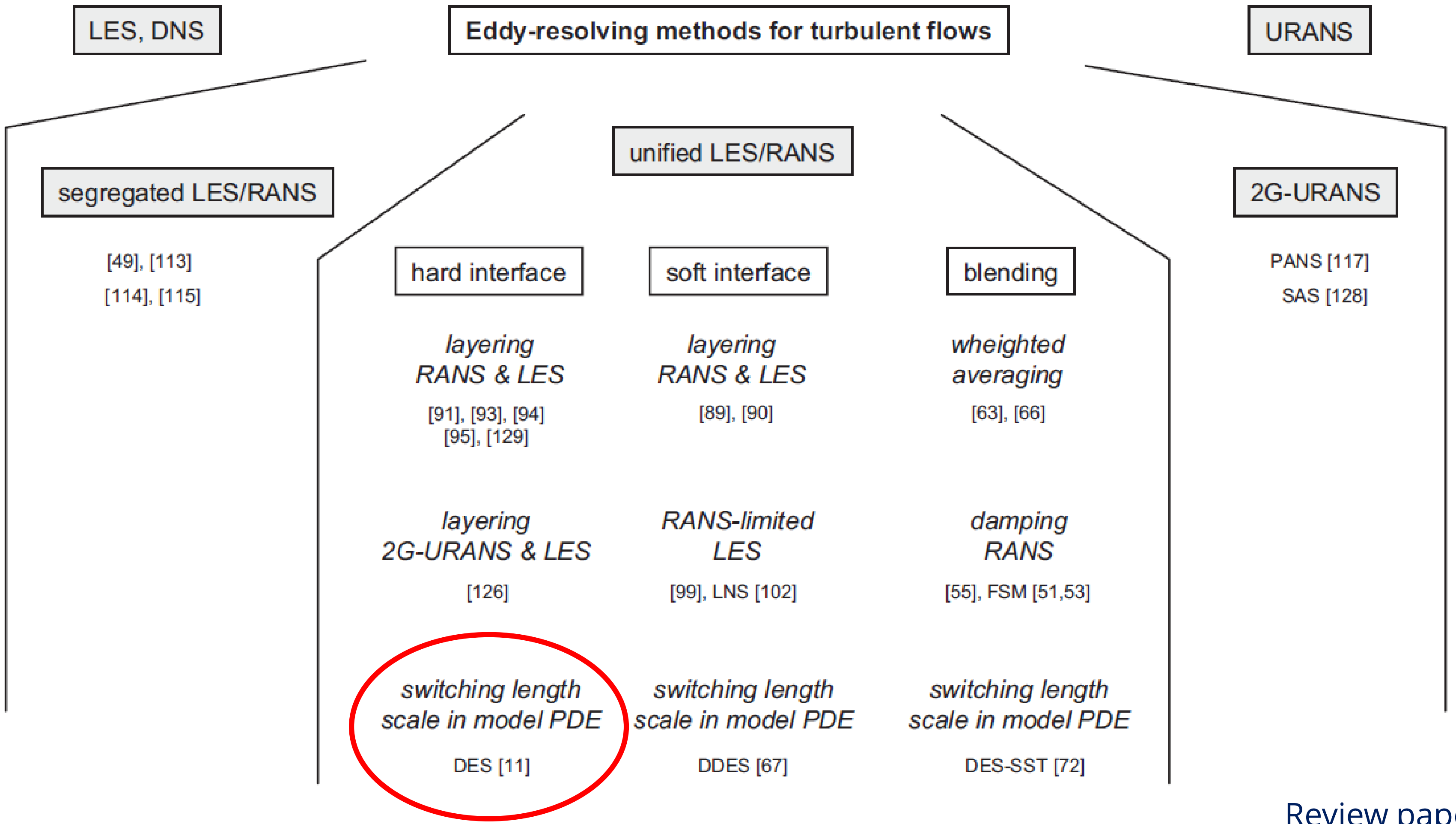
with Δ step size of computational grid and constant C_s

Many other EVMs and other multi-scale approaches [Sagaut 2006, Fröhlich 2006]

Some methods use **modified RANS models** to get ν_t → **hybrid models**

Example: Spalart Allmaras model with Δ as length scale → „Detached Eddy Simulation“ (DES) [Spalart 2009]

Classification of eddy-resolving methods (also called scale-resolving methods)

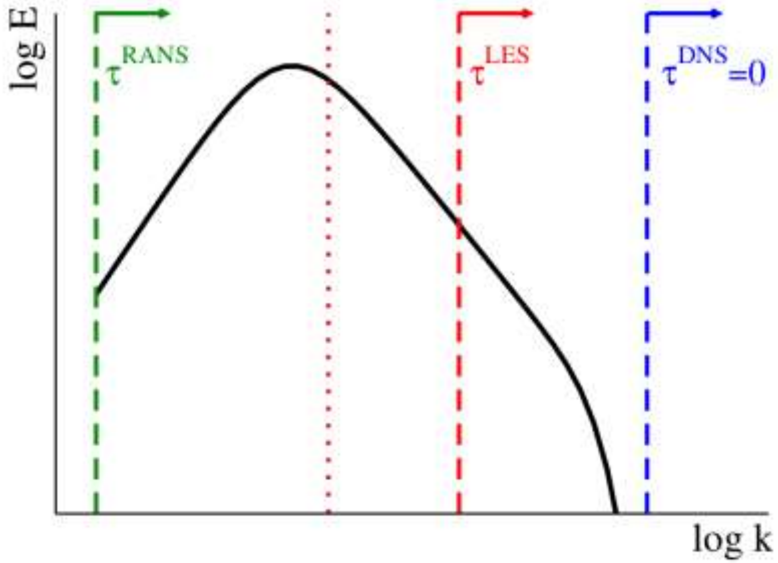


Review paper on hybrid methods
[Fröhlich, von Terzi 2008]

Resolution

LES

→ cutoff in inertial range



[Fröhlich, von Terzi 2008]

hybrid models

→ often cutoff at larger scales

→ risky, sufficient amount of resolved TKE needed

RANS model in in unstready mode (URANS) for self-generated unsteadiness (e.g. vortex shedding) **not recommended.**

Dissipation by model must be reduced → better use true hybrid (DES, SAS, etc.)

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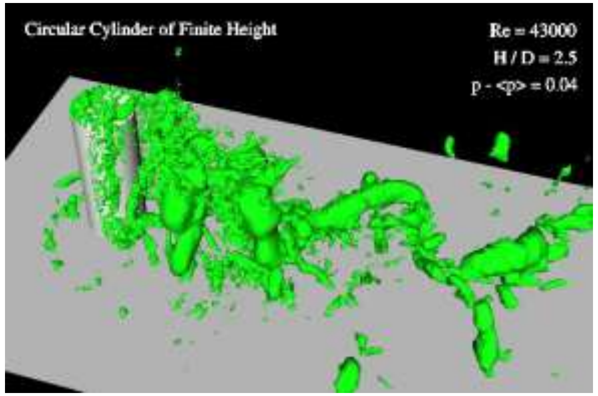
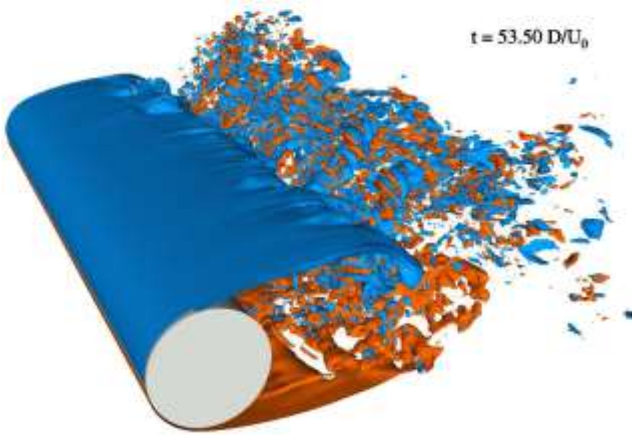




Vortex detection

What is a vortex?

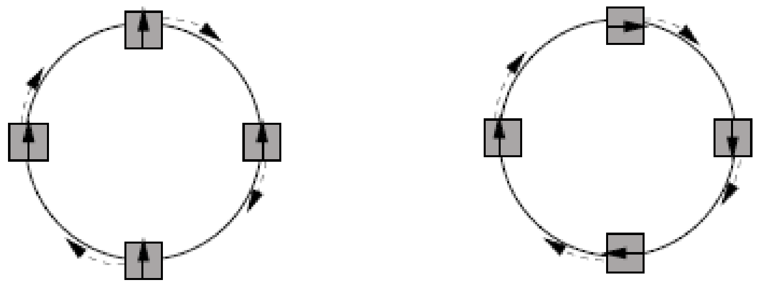
- Community has not agreed on exact definition
- Rule of thumb: „some kind of circular streamline“

- Vorticity $\omega = rot(u) = \nabla \times u$
  - Rotation of infinitisimaly small fluid element
  - NOT a criterion for a vortex

    1) possible to have circular streamline but $\omega = 0$ (e.g. potential vortex)

    2) possible to have straight streamlines (no vortex) with $\omega \neq 0$

    e.g. laminar Poiseulle flow in straight duct



$\omega = 0$ $\quad\quad$ $\omega \neq 0$

→ Do NOT use vorticity to detect vortices

The name is just missleading

Vortex detection

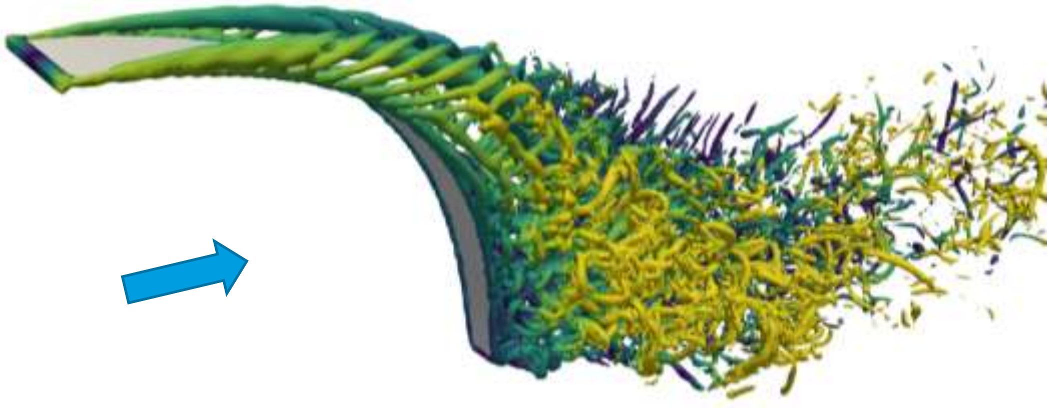
Rule of thumb: „some kind of circular streamline“

- But streamlines depend on velocity of observer, i.e. mean flow
- Further requirements (e.g. independent of coordinate system) → use velocity gradients only
- Vast number of different criteria
- Good overview in [Epps 2017]

Here only selected Eulerian criteria

Basic idea:

- Define scalar quantity $C(x,t)$in whole flow field
- Define a threshold C_{crit}
- Identify all points $C > C_{crit}$ as part of a vortex (or $C < C_{crit}$)
- Draw level surface of points where $C = C_{crit}$ → vortex shape



Color = distance from center plane

[Schoppmann et al. 2021]

Pressure minimum

Idea: rotation → centrifugal forces → pressure minimum

<u>Naive Procedure</u>

- set $C = p$
- select $p_{crit} < 0$
- draw iso-surfaces of p_{crit}

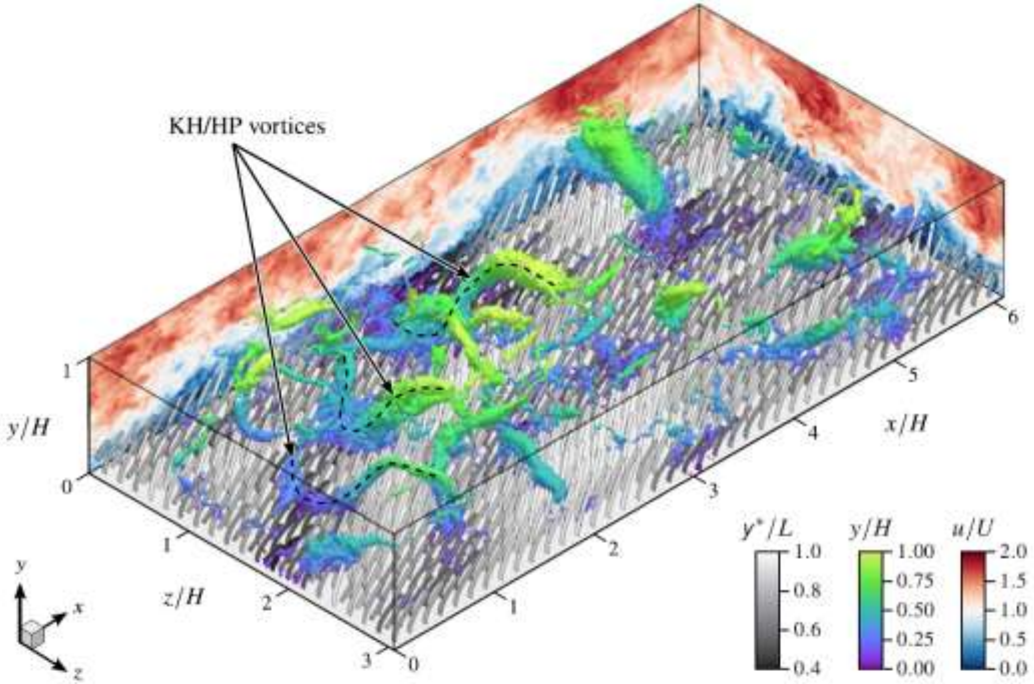
Problem: if mean pressure changes in different regions of flow, universal threshold p_{crit} does not work

<u>Better Procedure</u>

- set $C = p' = p - \overline{p}$
- select $p'_{crit} < 0$ (can use stagnation pressure as reference)
- draw iso-surfaces of p'_{crit}

Very simple

Gives large-scale vortices (cf. below)



$p'_{crit} \,/\, (0.5\,\rho U^2) = -0.4$

[Tschisgale et al. 2021

The λ_2 - criterion [Jeong, Hussain 1995]

Basis in velocity gradient tensor $$G_{ij} = \frac{\partial u_i}{\partial x_j}$$

Decomposition
$$G_{ij} = \underbrace{\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)}_{\text{strain rate tensor}} + \underbrace{\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right)}_{\text{rotation rate tensor}} = S_{ij} + \Omega_{ij}$$

Transport equation for S_{ij} neglecting unsteady and convective term, neglect viscous term

$$S_{ik}S_{kj} + \Omega_{ik}\Omega_{kj} = -\frac{1}{\rho}\frac{\partial^2 p}{\partial x_i \partial x_j}$$

If pressure Hessian $\frac{\partial^2 p}{\partial x_i \partial x_j}$ has two negative eigenvalues → pressure minimum (e.v. are real as tensor symmetric)

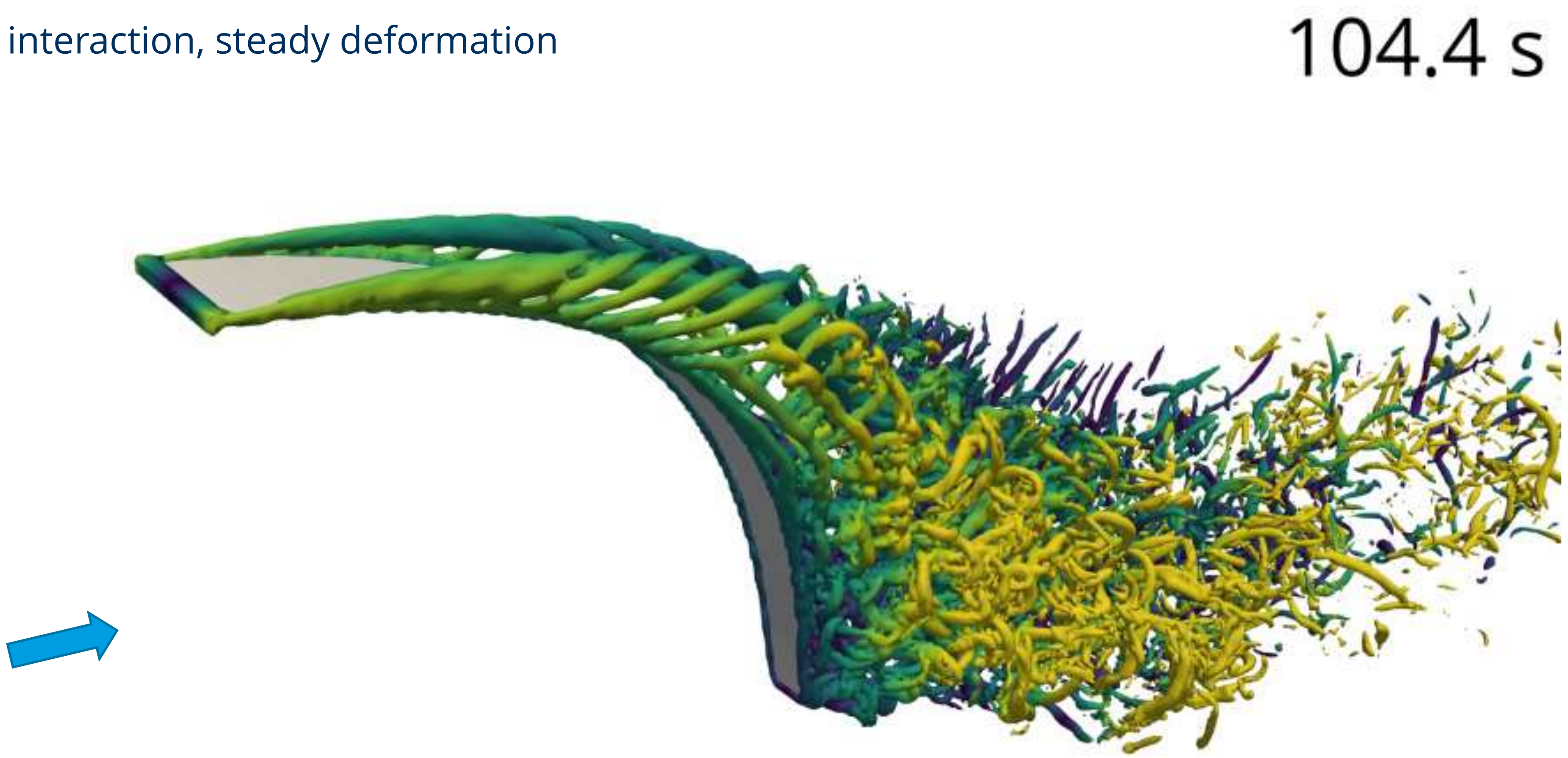
Procedure

- compute $S_{ik}S_{kj} + \Omega_{ik}\Omega_{kj}$
- compute eigenvalues $\lambda_1 \geq \lambda_2 \geq \lambda_3$. If $\lambda_2 < 0$ two negative eigenvalues
- set $C = \lambda_2$ and choose $C_{crit} < 0$ (value usually selected manually)

The λ_2 - criterion at work

Flow around deformable trapezoidal ribbon

→ Fluid-structure interaction, steady deformation

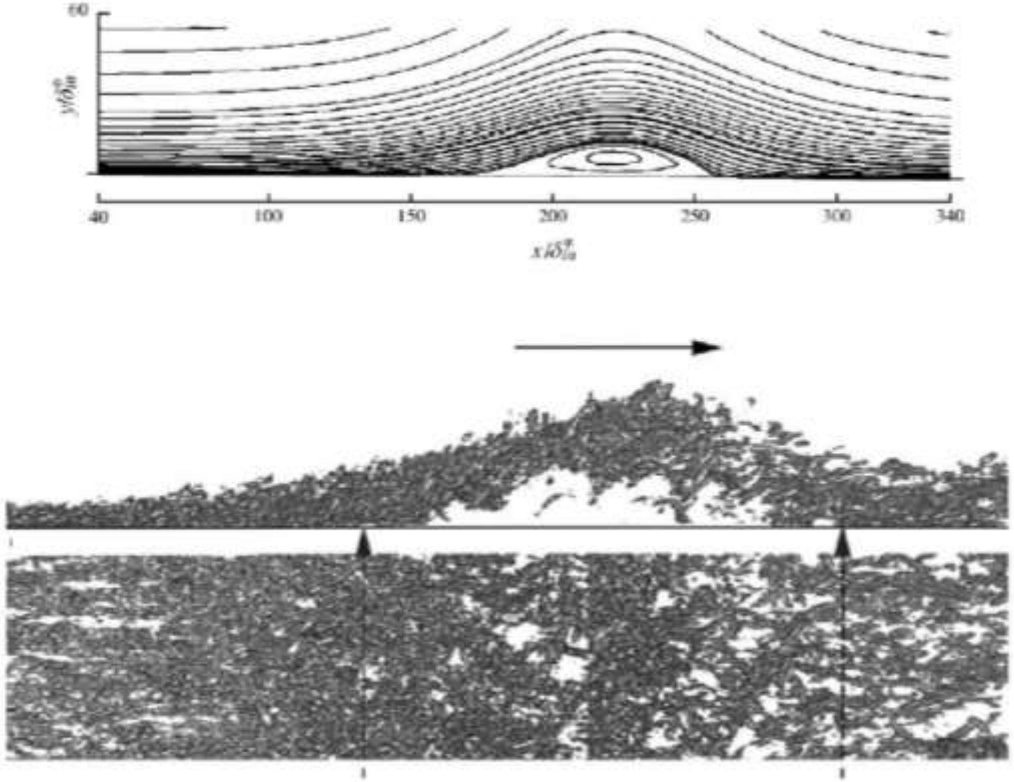


Color = distance from center plane

[Schoppmann et al. 2021]

Often just „snow“ with vortex criteria

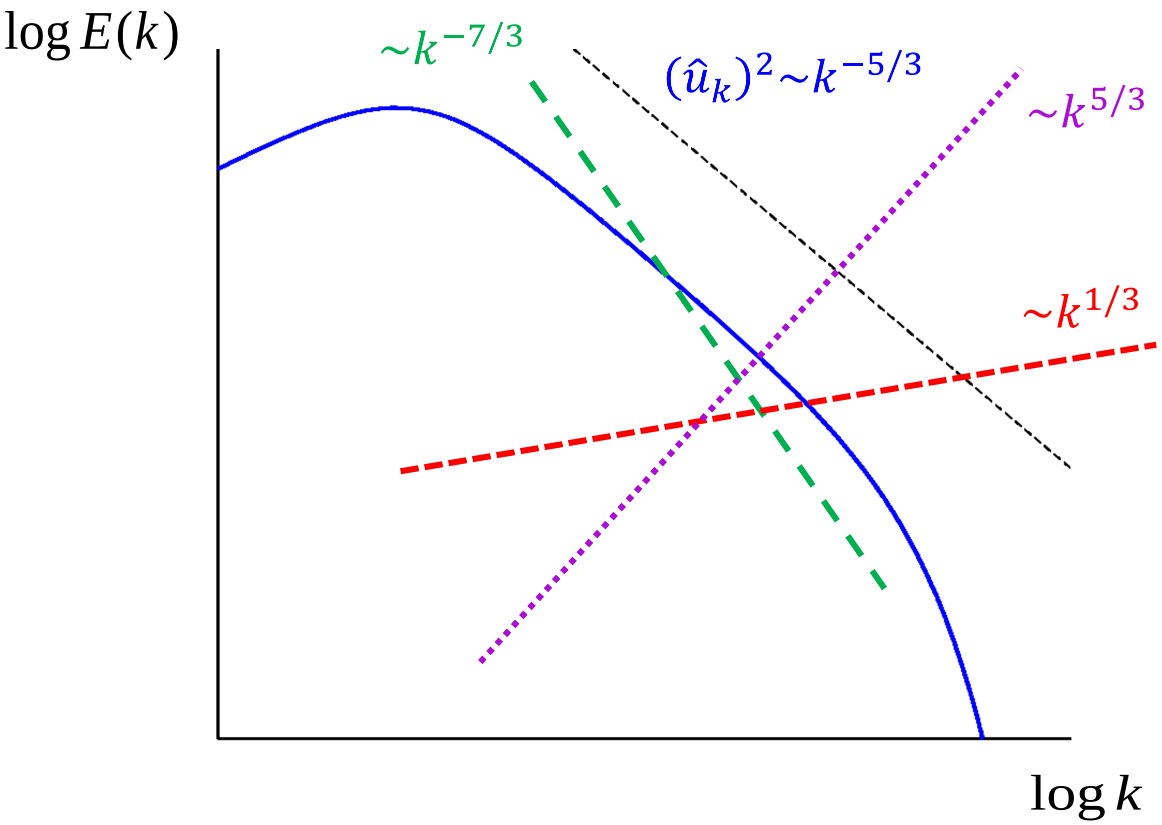
Discriminant criterion D (similar approach as λ_2) [Chong et al. 1998]



[Na, Moin, 1998]

Spektral behaviour of criteria

quantity	„energy spectra“ (square of Fourier coeff.)
p	$\mathcal{E}_p \sim \varepsilon^{4/3}\ k^{-7/3}$
u	$\mathcal{E}_u \sim \varepsilon^{2/3}\ k^{-5/3}$
ω	$\mathcal{E}_\omega \sim \varepsilon^{2/3}\ k^{1/3}$
Q	$\mathcal{E}_Q \sim \varepsilon^{4/3}\ k^{5/3}$
λ_2	$\mathcal{E}_{\lambda_2} \sim \varepsilon^{4/3}\ k^{5/3}$
D	$\mathcal{E}_D \sim \varepsilon^2\ k^3$
Dissipation	$\mathcal{D} \sim k^{1/3}$



p → extracts larger scales than u
ω → smaller scales than u
λ_2 → much finer scales than u
Q → much finer scales than u
D → extremly fine scales

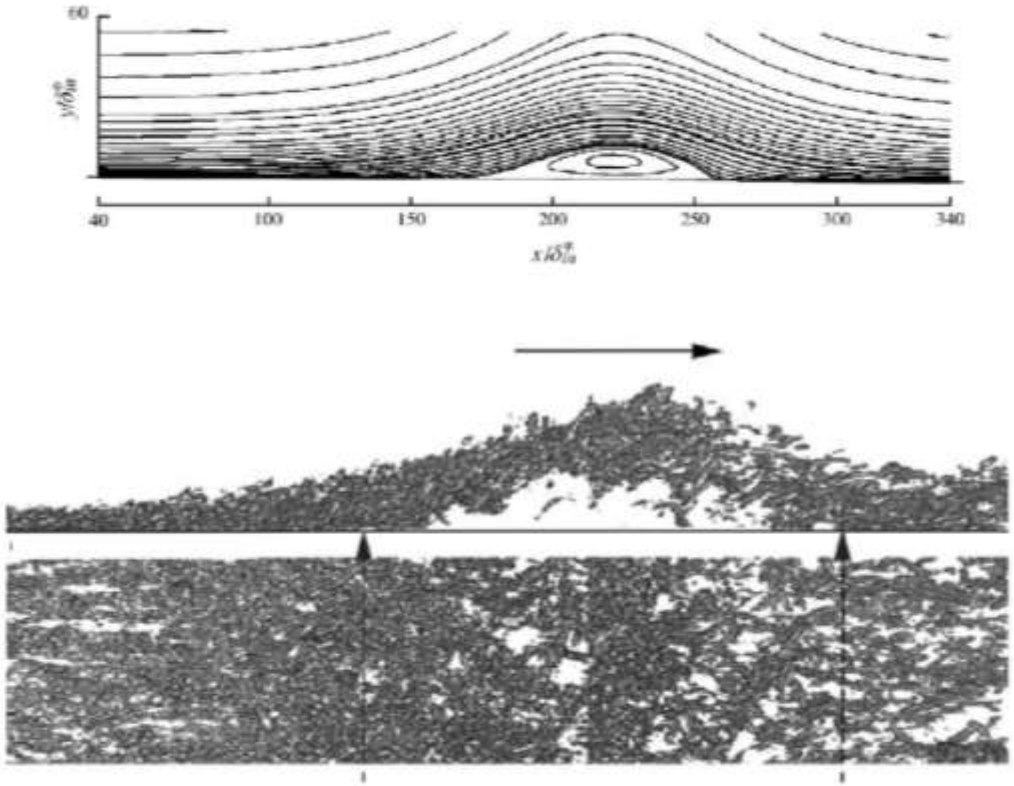
} Increasing influence of numerical errors on structures extracted

→ Different criteria select different sizes of vortex structures !

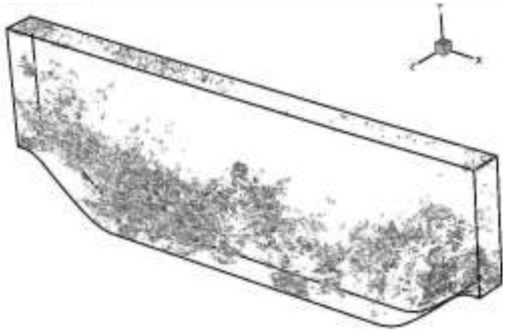
[Fröhlich, 2006, p.291]

Often just „snow"

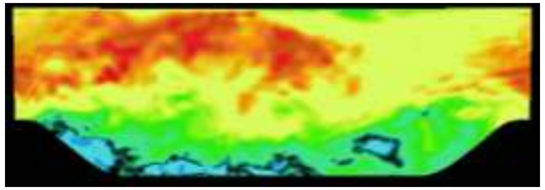
Discriminant criterion D (another Eulerian criterion) [Chong et al. 1998]



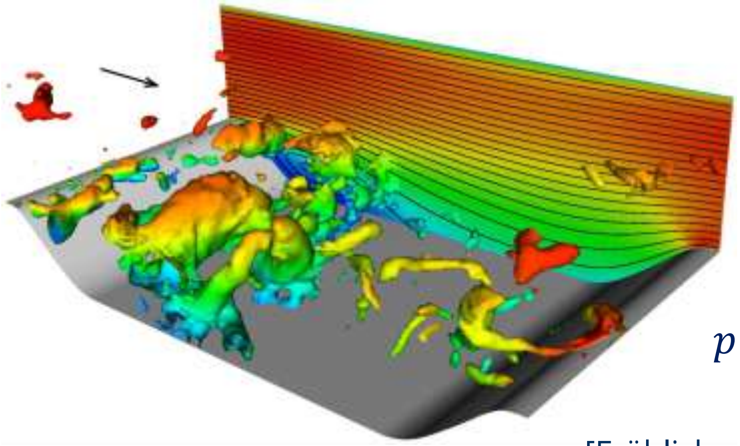
[Na, Moin, 1998]



D



u



$p - \langle p \rangle_t$

[Fröhlich et al. 2005]

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Final word

Select models suitable for given flow

Wintergerste, Casey, Best Practice Guidelines, ERCOFTAC, 2000, ISBN - 978-0-9955779-2-3

https://www.ercoftac.org/publications/ercoftac_best_practice_guidelines/

Always do serious validation

grid resolution study

size-of-domain study

sensitivity to choice of model for turbulence

comparison for benchmark cases similar to given flow

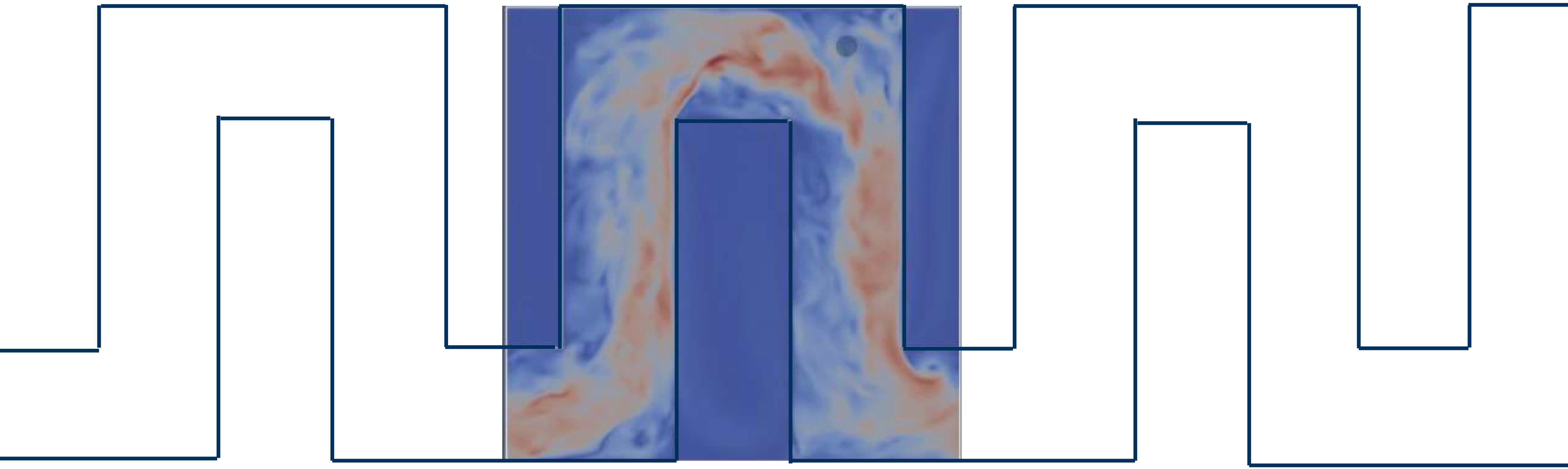
…

Data Bases

https://turbmodels.larc.nasa.gov/ → lots of benchmarking data for aerodynamics applications

https://kbwiki.ercoftac.org → very good data base for wide range of flows

Have fun with turbulence !



[Hafemann, Fröhlich, ISM]

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