Mesh Generation

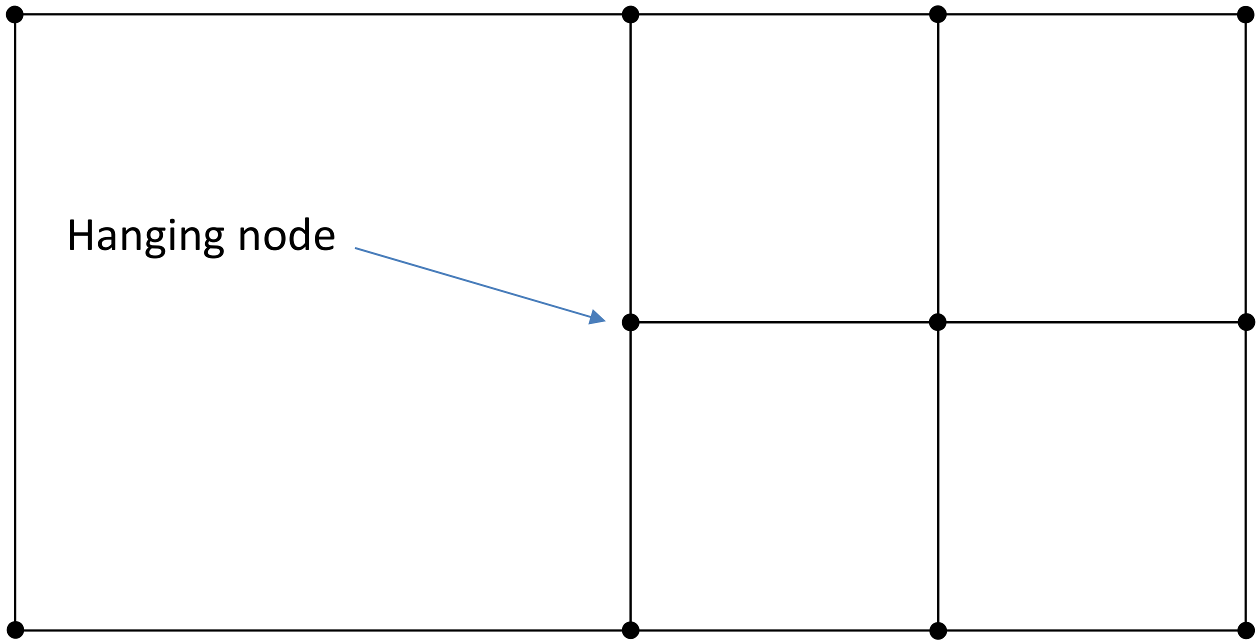
Outline

- Introduction
- Numerical methods
- Element shape
- Structured 2D meshes
- Unstructured 2D meshes
- Hexahedral meshes
- Tetrahedral meshes

Introduction

- Mesh
  - Discretization of a geometric domain into small (simple) shapes
    - Elements or cells
      - Triangles, quadrilaterals (two dimensions)
      - Tetrahedra, hexahedra (three dimensions)

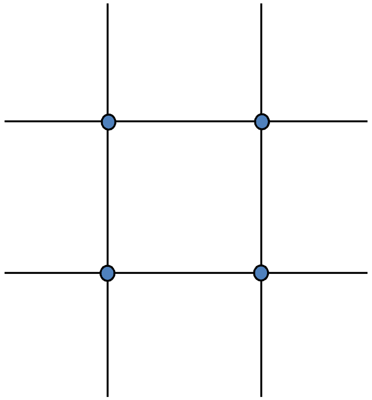
Conforming vs. Non-conforming Meshes



Computational Mesh

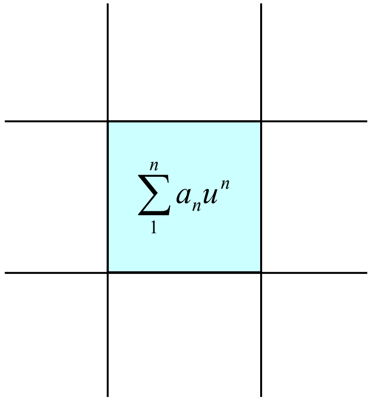
- Numerical computations require discretized space
- Mesh differently interpreted, depending on numerical scheme:

Finite Differences



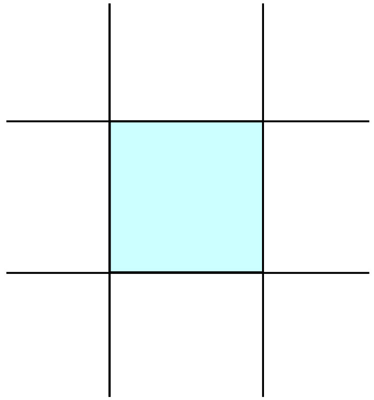
Values are stored on the Mesh-nodes

Finite Volume



Mesh-elements are used as control volumina

Finite Elements



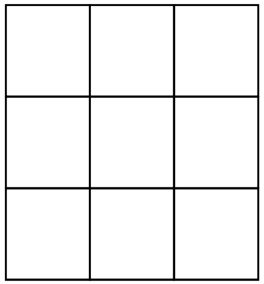
Mesh-element define an area in which a function is valid

Types of Meshes

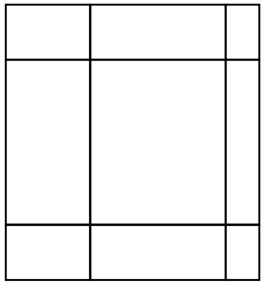
- Structured mesh
  - All interior vertices are topologically alike
- Unstructured mesh
  - Vertices may have arbitrarily varying local neighborhoods
- Block-structured
  - Formed by a number of small structured meshes combined in an overall unstructured pattern
- Hybrid
  - Formed by a number of small structured and unstructured meshes combined in an overall unstructured pattern

Structured Meshes

- Advantages
  - Simple
  - Efficient
    - Memory/execution time
      - Neighboring cells are given implicitly
        - » *Roughly factor three memory savings compared to unstructured meshes*



equidistant



non equidistant

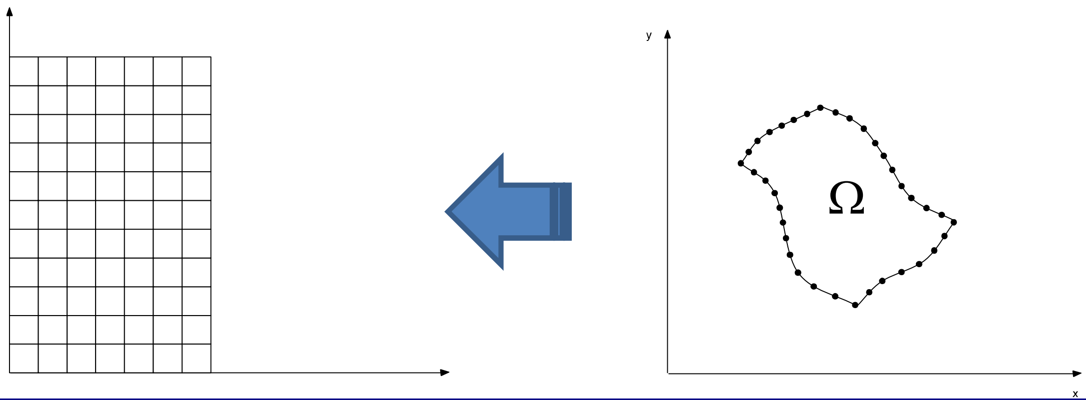
Structured Meshes

- Disadvantages
  - Complex or impossible to generate structured meshes for complex geometries
  - Structured meshes may require many more elements because elements cannot grade in size rapidly

- Always a tradeoff

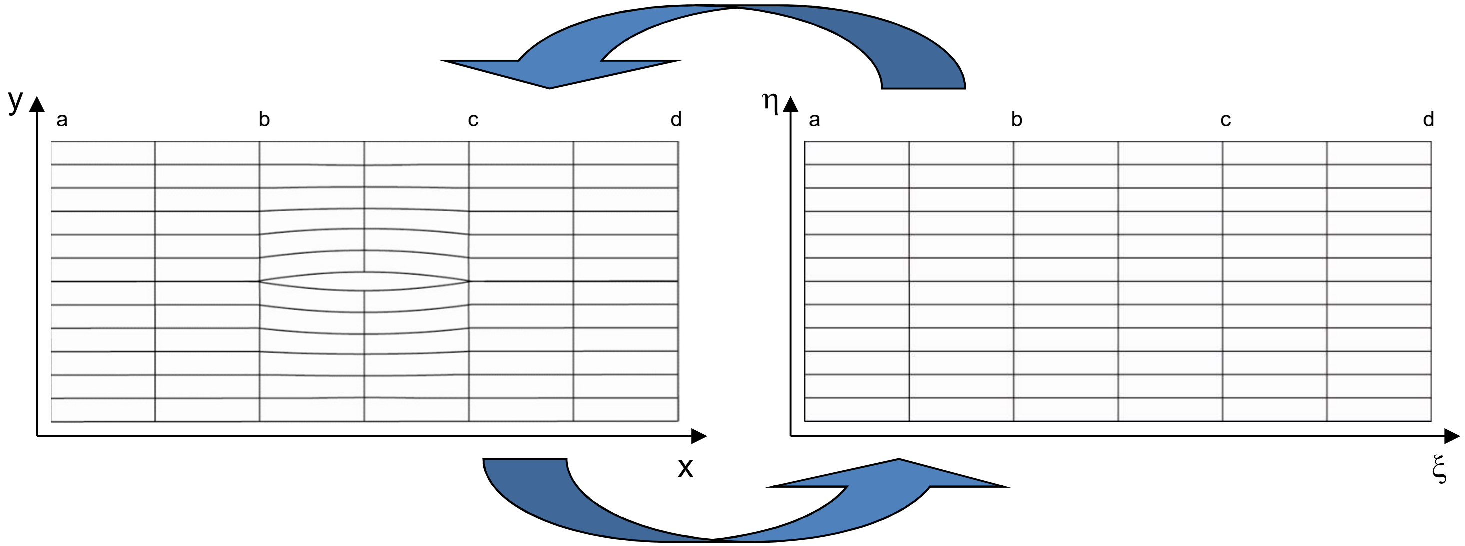
Structured Meshes

- Generation
  - Hand-generated
  - Algebraic
  - PDE
    - Mapping of domain Ω to an other domain with a convenient coordinate system



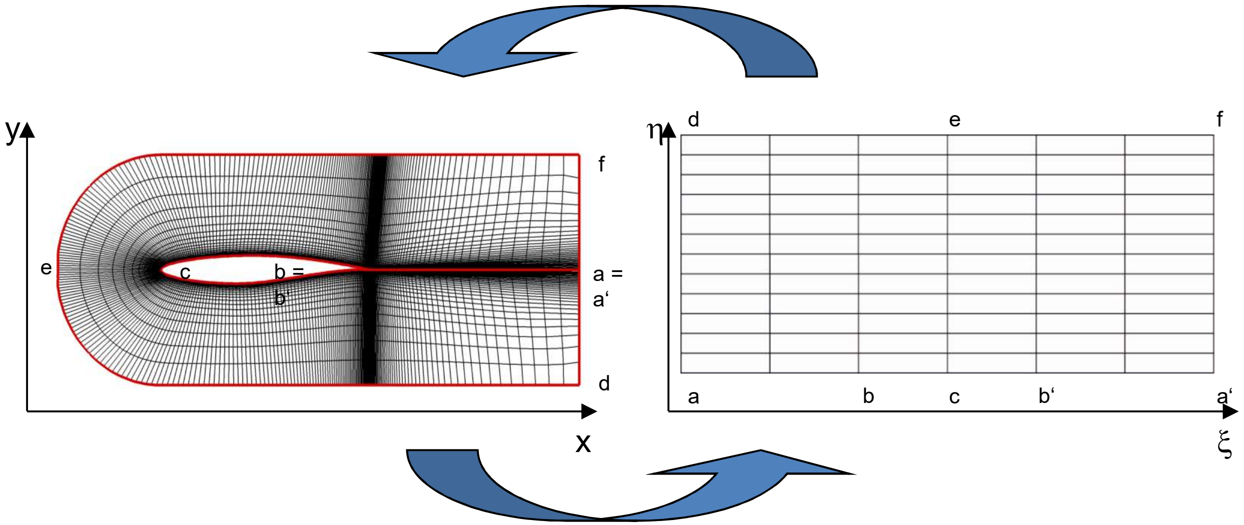
Bodyfitted Structured Meshes (H-Grid)

- Idea: Skewing of the cartesian mesh, so that more complex geometries can be discretized.
- For the numerical calculation the skewed cartesian mesh x-y is mapped onto a unskewed mesh ξ-η
- The geometry is limited by two mesh lines



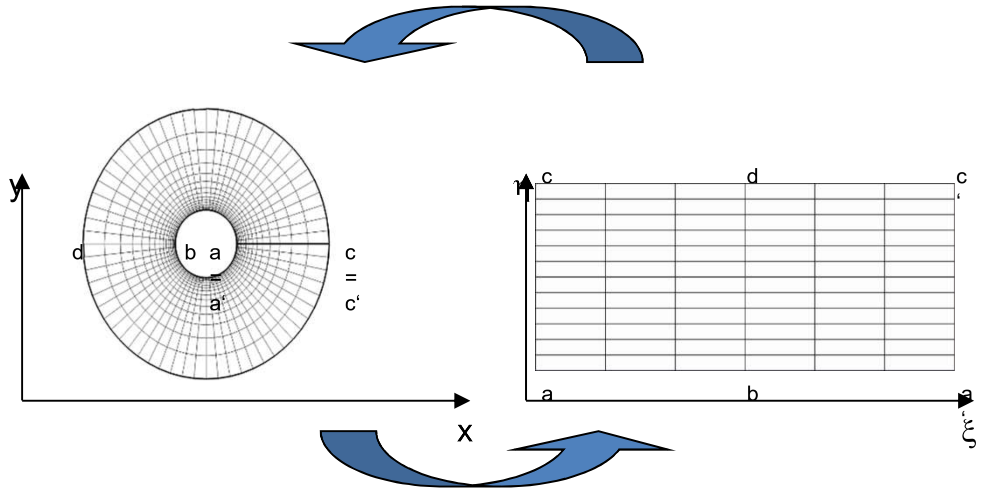
Bodyfitted Cartesian Mesh (C-Netz)

- Idea: The mesh is skewed in a fashion, that it wraps around the geometry
- The geometry is confined by one single mesh line



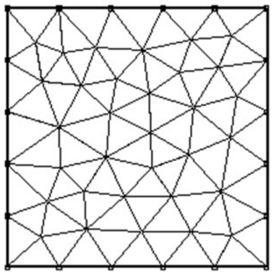
Bodyfitted Cartesian Mesh (O-Mesh)

- Idea: The mesh is skewed in a fashion, that it wraps around the geometry
- The geometry is confined by one single mesh line



Unstructured Meshes

- Advantages
  - Flexibility in fitting complicated domains
  - Rapid grading
  - Relatively easy refinement



Unstructured Two-dimensional Meshes

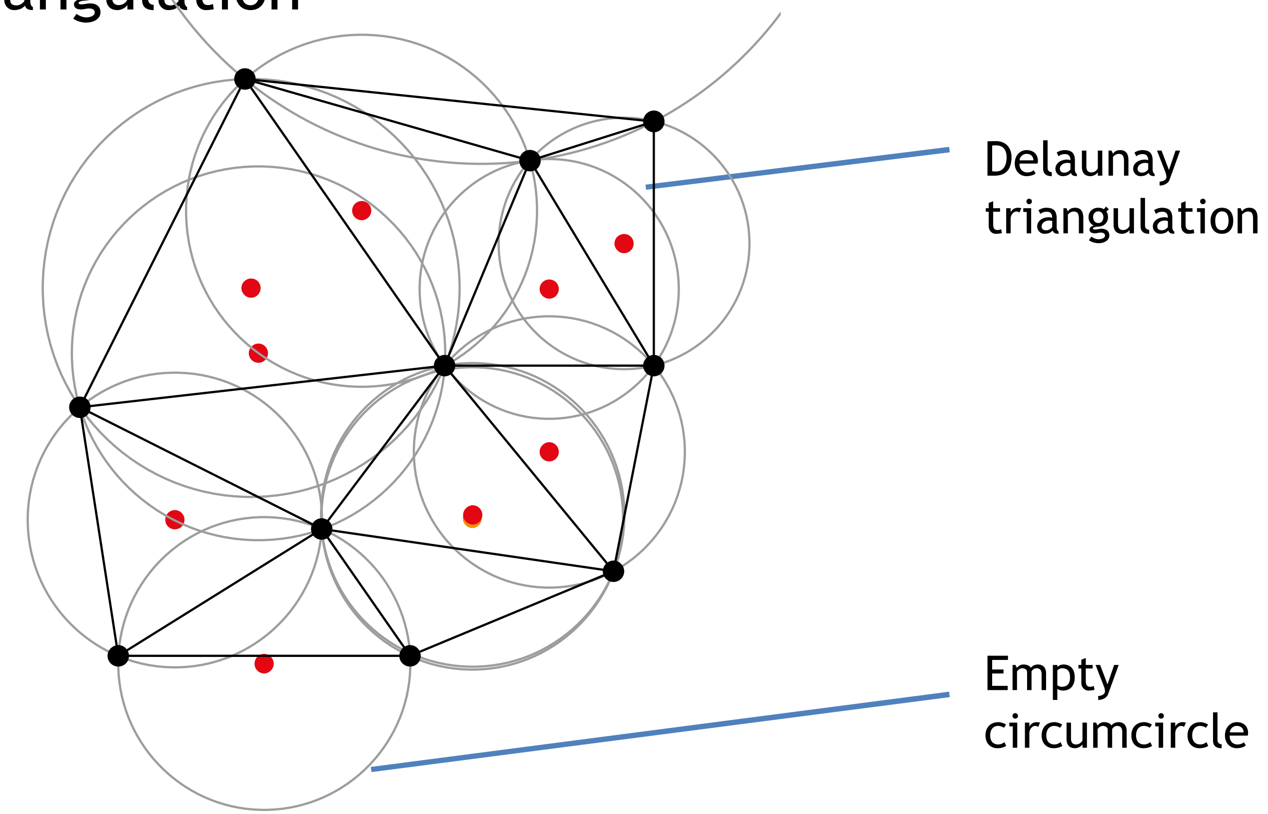
- Generation
  - Delaunay Triangulation
  - Quadtrees

Generating Unstructured Meshes

- Delaunay Triangulation
  - Find non-overlapping triangles, that fill the convex hull of a set of points
  - Properties:
    - Every edge is shared by at most two triangles
    - The circumcircle of a triangle contains no other input points
    - Maximizes the minimum angle of all the triangles

Example in 2D

- Delaunay Triangulation



Source: https://en.wikipedia.org/wiki/Delaunay_triangulation

Generating Unstructured Meshes

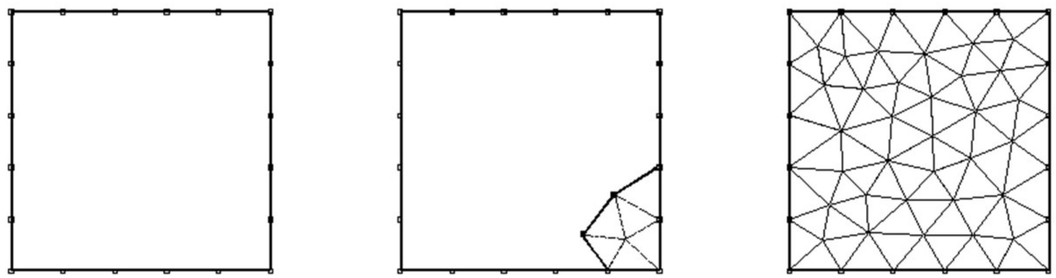
- Delaunay Triangulation
  - General procedure:
    - Placement of mesh vertices
    - Triangulation

Generating Unstructured Meshes

- Delaunay Triangulation
  - Placement Phase:
    - Vertices around the boundaries first
      - Mostly fixed by user
    - Generation of internal vertices
      - Advancing front
      - Interpolation from coarse background triangulation
        - » *"random" placement*

Example Algorithm

- Advancing Front Algorithm
  - Discretize the boundary as initial front
  - Add elements into the domain and update the front
  - When front is empty, the mesh is complete

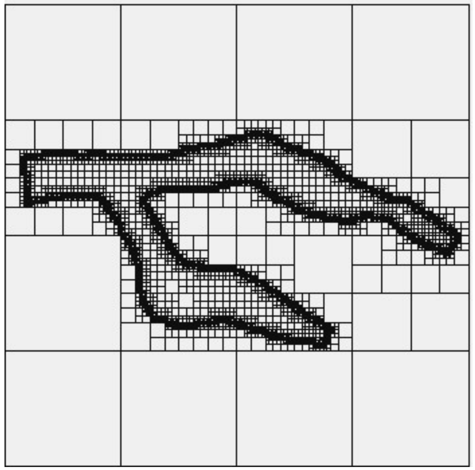


a) Initial Front   b) Advancing the front   c) Final mesh

Source: http://www.ae.metu.edu.tr/~cengiz/thesis/afm.html

Spatial Search

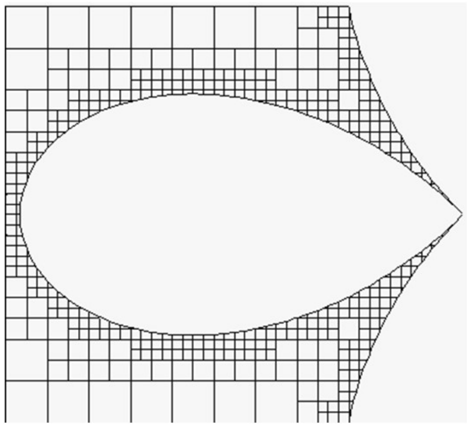
- Quadtrees
  - Recursive subdivision from an initial root square
  - Per definition non-conforming
  - Staircase approximation of geometry
    - Further treatment necessary
  - Often used as background mesh
    - E.g. Delaunay algorithm



Source: https://www.staff.ncl.ac.uk/qiuhua.liang/Research/grid_generation.html

Boundary Approximation

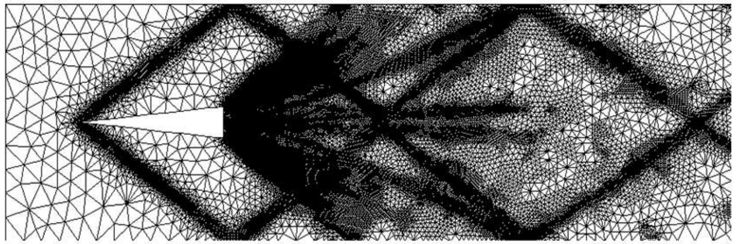
- Quadtree geometry approximation methods
  - Cut cells/deform cells
  - Numerically handled within the solver
    - Lattice Boltzmann (q-values)
    - Discontinuous Galerkin (penalty terms)



Source: https://www.staff.ncl.ac.uk/qiuhua.liang/Research/grid_generation.html

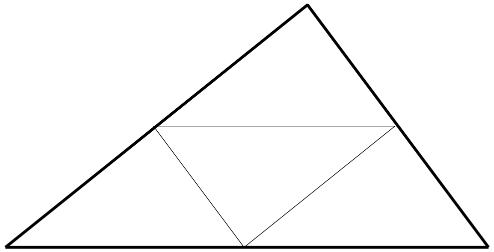
Unstructured Mesh Refinement

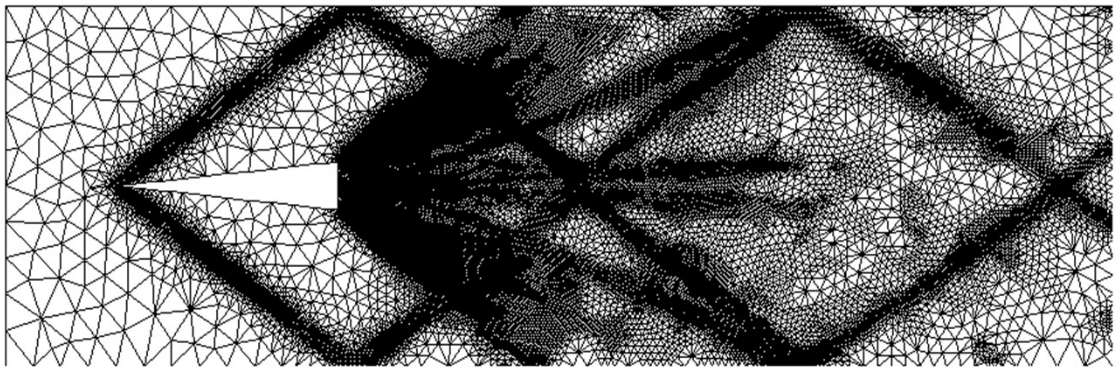
- Mesh Refinement
  - Two general flavors
    - Static mesh refinement
      - Taken care of during initial mesh generation
    - Adaptive mesh refinement
      - Areas in which the error of the PDE solution is large can be refined during simulation
        - » *Solver feature not one of the mesh generator*
      - mostly triggered by indicators
        - » *E.g. pressure gradient*



2D Illustration for Mesh Refinement

- Example for mesh refinement
  - Subdivision of the original mesh triangle into four smaller ones



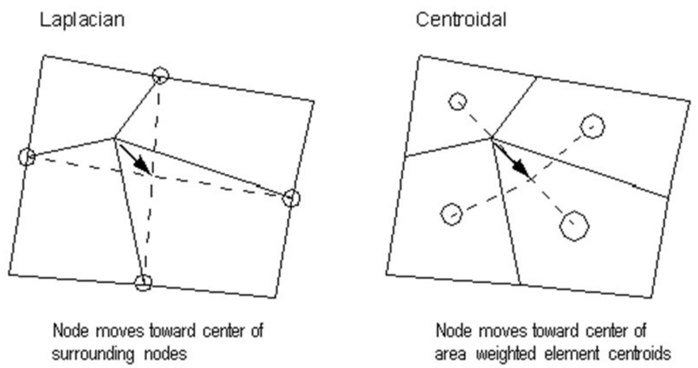


Generating Unstructured Meshes

- Mesh improvement
  - Smoothing
    - Improve element shapes
    - Adjusts the locations of mesh vertices
    - Topology stays invariant

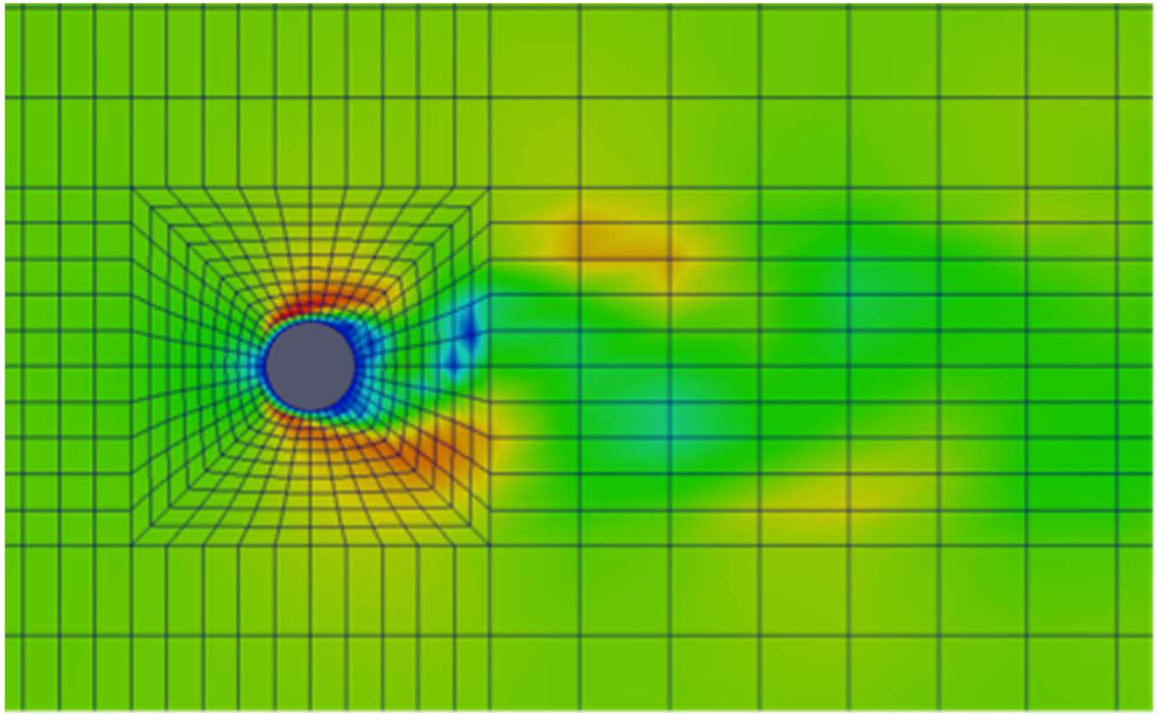
Mesh Smoothing Illustration 2D

- Mesh improvement
  - Smoothing
    - Laplacian smoothing
      - Sweeps over the entire domain multiple times
      - Moves each adjustable vertex to the arithmetic average of the vertices adjacent



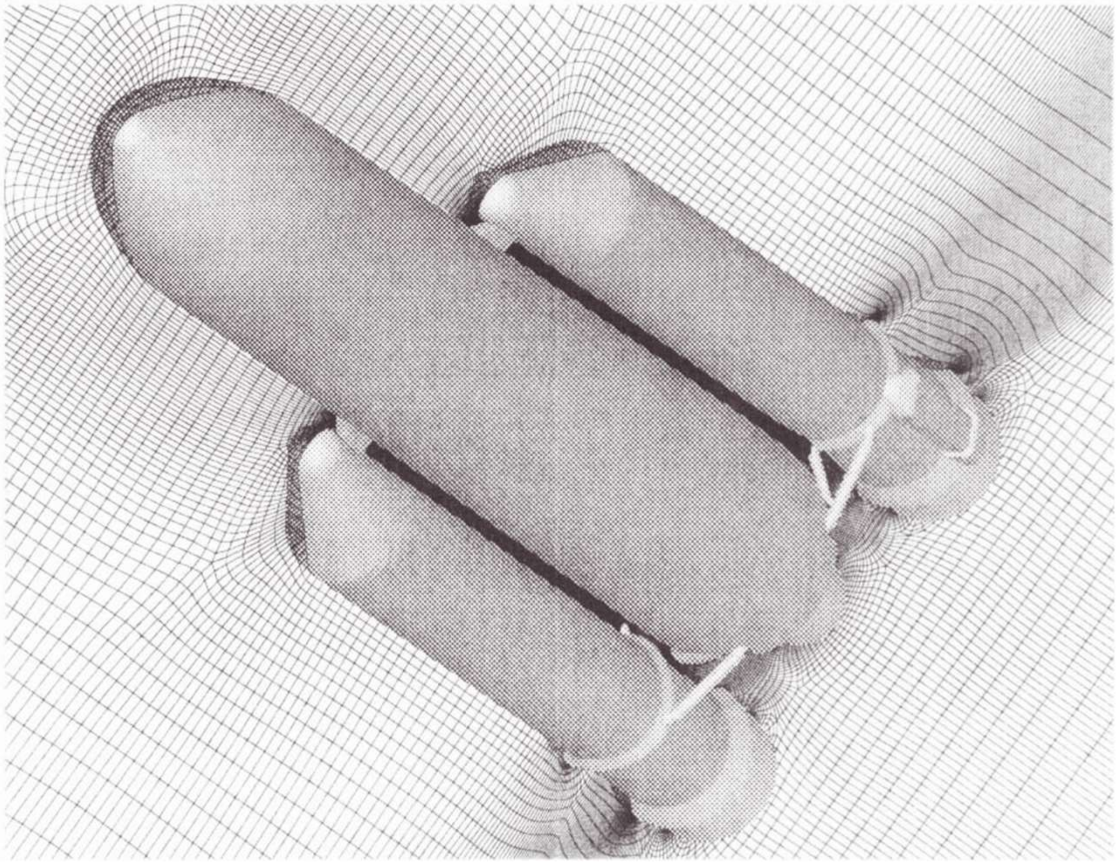
Source: http://docs.salome-platform.org/latest/gui/SMESH/smoothing_page.html

Block-structured Meshes



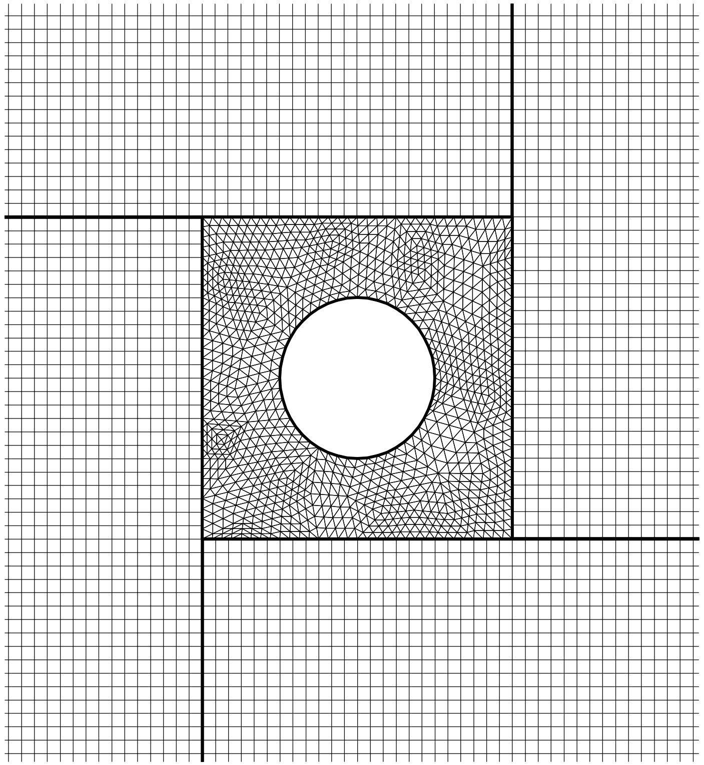
Source: http://bsccase02.bsc.es/alya/tutorial/case_cylinder.html

Multi-Block Structured Meshes



Source: https://spinoff.nasa.gov/spinoff2002/ct_3.html

Hybrid Meshes

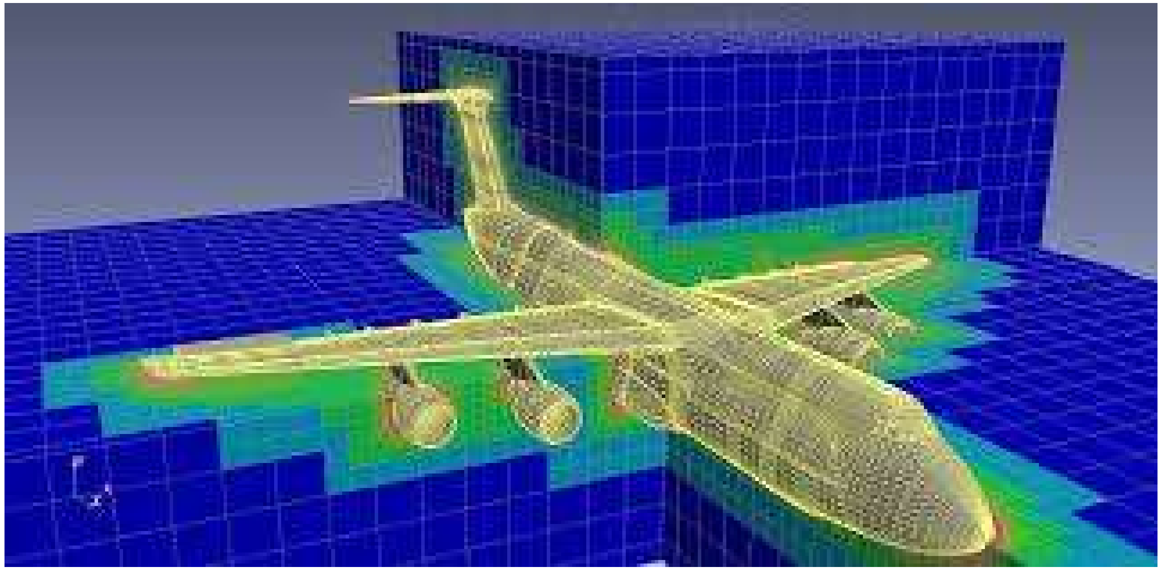


Hexahedral Meshes

- Multiblock Meshes
- Cartesian Meshes
- Unstructured Hexahedral Meshes

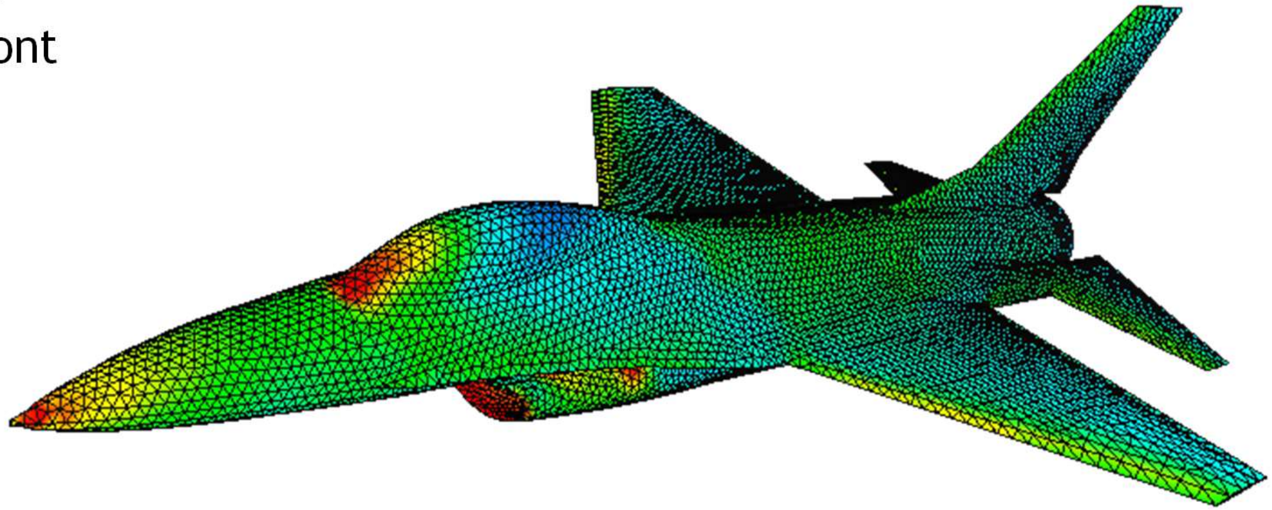
Cartesian Meshes

- Octree meshes
  - An initial bounding cube is recursively split in each dimension
  - Results in eight congruent cubical child elements in 3D



Tetrahedral Meshes

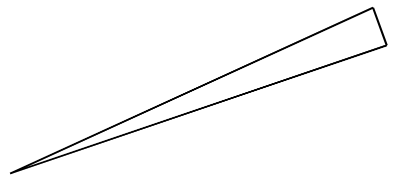
- Great flexibility in fitting complicated domains
- Easy refinement
- Generation
  - Delaunay triangulation in 3D
    - Advancing Front
    - Octree



Source: http://www.aerosoftinc.com/gasp/gallery.php

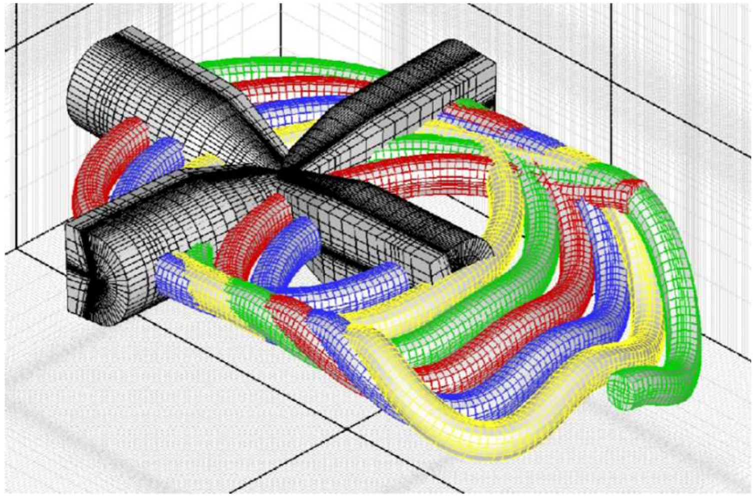
Quality Criteria of Meshes

- Quality of mesh influences numerical results directly
  - Solution quality
    - E.g. tetrahedrons in boundary layer
  - Running time
    - E.g. small or misshaped element that reduces the global possible time-step



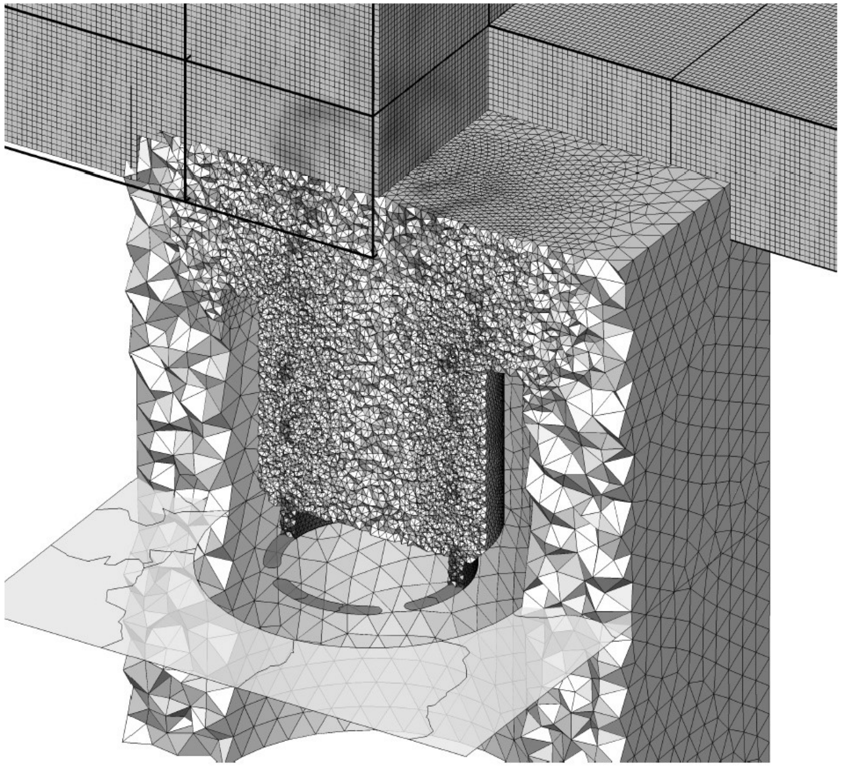
Chimera

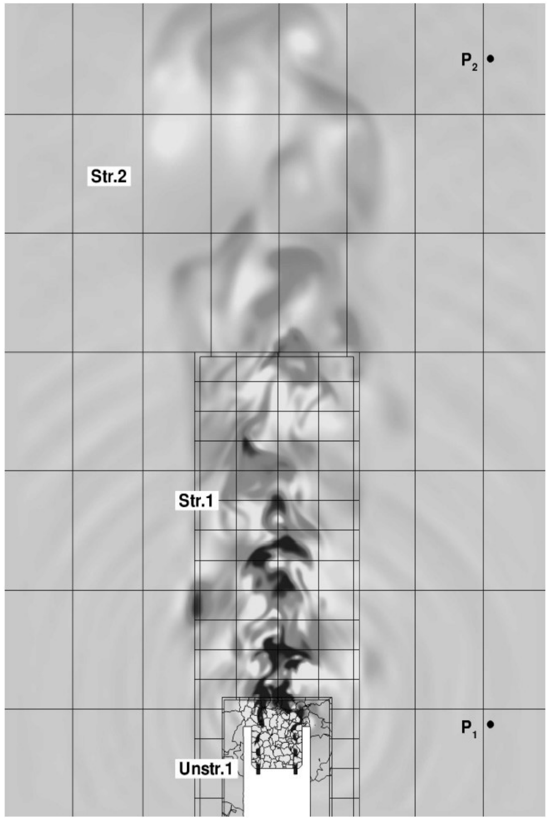
- Often it is too complicated to produce useful meshes, when more than one geometry is involved.
- Also the simulation for rotating geometries is very difficult



Chimera

Examples





Outtakes